A Finite - Difference Time-Domain Technique for Analysis Nonlinear Waveguide Structure

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الملخص

في هـــذا البحث قمنا بتطبيق طريقة الفروق المتناهية بالمجال الزمني الصريحة(Explicit FDTD) لحــــل المعادلات الموجية غير الخطية(Nonlinear Wave Equation)، وكان مجال الحل لهذه المشـــكلة هو عبارة عن مستطيل مقسم إلى أربعة طبقات موجية واحدة منها غير خطية. وقمنا كذلك بدراسة وتحليل ثبات (Stability) هذه الطريقة ووضع الشروط المناسبة لتجعلها طريقة مستقرة وفعالة تقودنا للحل المظبوط .

Abstract

In this paper an explicit finite-difference time-domain (FDTD) approach for solving the wave equation in nonlinear optical waveguiding rectangular structure is proposed. The validity of the proposed technique is demonstrated through its applications on a rectangular four layers waveguide structure where one layer is a nonlinear medium. The stability conditions are also derived and tested.

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Introduction

The Finite-Difference Time-Domain (FDTD) scheme [1,2] has shown a great popularity for high frequency electromagnetic problems, because it is an efficient and easy to program. The advantages of these techniques are in terms of its accuracy, generality, computational efficiency, and ease of use. Since the optical waveguides are defined by a two-dimensional distribution of the index of refraction, the most accurate methods must treat the full (2-D) problem. Unfortunately, the computer memory and time requirements are such that it is impossible to solve a realistic (2-D) problem on a PC or workstation. As a consequence, the problem is usually reduced to a somewhat equivalent one-dimension problem. The continuous exploration of new optical device concepts for applications in communications and signal processing has been recently leading to possibility of using the nonlinear property of materials. Examples of nonlinear devices are switches, couplers, beam steerers, and logical elements [3-6]. Nonlinearity provides the designer an added degree of design flexibility. With the growing complexity of nonlinear device structures, approximate analytical techniques become inadequate. So more accurate, and efficient numerical techniques are sought. In the past few years the Finite-Difference Time-Domain (FDTD) method has emerged as one of the most versatile numerical methods in optical waveguide analysis because it can readily incorporate almost any type of constitutive relation describing the medium in use, including nonlinear media. By directly solving Maxwell's equations simultaneously with the medium constitutive relation in the time domain, the method fully accounts for the effects of reflection, diffraction, radiation, and suitable nonlinear effects that cannot otherwise be predicted by approximate analytical or other numerical techniques. A simple application of the FDTD method to nonlinear media employs an explicit time-stepping scheme in the discretization where the nonlinear permittivity at the unknown time step $N+1$ is approximated using the electric-field value at the current time step N [3,7]. The scheme allows the unknown field at time step $N+1$ to be explicity solved for, but it places a severe stability constraint on the time step and mesh size. In some cases, the time step has to be reduced 20-30 times below the limit set by the Courant-Fridrichs-Lewy (CFL) condition to achieve stability in the nonlinear medium regions [7]. On a more fundamental level, the above scheme introduces an artificial time leg equal to the time step Δt in the medium response because the electric displacement D at the time step $N+1$ is computed using the electric field E at the same time step and permittivity value at the previous time step N. To eliminate this artificial time leg, Josef and Tavlove proposed an iterative solution of the nonlinear N maxwell equations [6,8]. In this paper , we extend this idea to general media characterized by instantaneous nonlinear permittivity ε and conductivity σ by introducing an explicit FDTD scheme. We also check the consistence , convergent and stability by using Fourior or Von Neomann method developed by J. Von Neumann [9].

Theory and discussion

The explicit FDTD method scheme yields a stable and efficient algorithm for solving Maxwell's equations in nonlinear media in the time domain. In a two-dimensional (2-D) waveguiding structure, the propagation of TE-polarized light is governed by a scalar-wave equation in terms of a single electric-field component E_Y as follows:

$$\mu_o \frac{\partial(\sigma E_y)}{\partial t} + \frac{1}{c^2}\frac{\partial^2(\varepsilon_r E_y)}{\partial t^2} = \frac{\partial^2(E_y)}{\partial x^2} + \frac{\partial^2(E_y)}{\partial z^2} \qquad (1)$$

In the above equation, c is the velocity of light in vacuum, σ is the electric conductivity, and ε_r is the relative permittivity. For nonlinear medium, ε_y is a function of the electric field. For instance, the relative permittivity of a material having instantaneous Kerr-type [10,11]nonlinearity which can be expressed as

$$\varepsilon_r = \varepsilon_r^L + \alpha\left|E_y\right|^2$$

where ε_r^L is the linear relative permittivity and α is the nonlinear coefficient. In the finite–difference time-domain method, the computational domain is partitioned into a grid of size Δx and Δz as seen in Figure (1).

The region in (x,y,t) space is covered by the rectangular grids parallel to the axes. The lines $x = x_i$, $y = y_j$ and $t = t_N$ are called the grid lines and their intersections are called the mesh points of the grid. At each interior mesh point (x_i, y_j, t_N) for $i, j = 1,2,...,k$ and $N = 1,2,...,M$, Eq. (1) is satisfied and we have

$$\mu_o \frac{\partial(\sigma E_Y)}{\partial t}(x_i, z_j, t_N) + \frac{1}{c^2}\frac{\partial^2(\varepsilon_r E_y)}{\partial t^2}(x_i, z_j, t_N) = \frac{\partial^2(E_y)}{\partial x^2}(x_i, z_j, t_N) + \frac{\partial^2(E_y)}{\partial z^2}(x_i, z_j, t_N) \qquad (2)$$

The simplest replacement of Eq. (2) consists of approximating the space and time derivative by central second order difference , and the forward difference for first time derivative is given by:

$$\frac{\partial E}{\partial t}(x_i,z_j,t_N)=\frac{E(x_i,z_j,t_N+l)-E(x_i,z_j,t_N)}{l}-\frac{l}{2}\frac{\partial^2 E}{\partial t^2}(x_i,z_j,\tau_N) \qquad (3)$$

where $\tau_N \in (t_N, t_N+l)$, $l=\Delta t$

The central difference for second space derivative with respect to x, z and t are,

$$\frac{\partial^2 E}{\partial x^2}(x_i,z_j,t_N)=\frac{E(x_i-h,z_j,t_N)-2E(x_i,z_j,t_N)+E(x_i+h,z_j,t_N)}{h^2} -\frac{h^2}{12}\frac{\partial^4 E}{\partial z^4}(\xi_i,z_j,t_N) \qquad (4)$$

where $\xi_i \in (x_i-h, x_i+h)$, $h=\Delta x$.

$$\frac{\partial^2 E}{\partial z^2}(x_i,z_j,t_N)=\frac{E(x_i,z_j-k,t_N)-2E(x_i,z_j,t_N)+E(x_i,z_j+k,t_N)}{k^2} -\frac{k^2}{12}\frac{\partial^4 E}{\partial z^4}(x_i,\zeta_j,t_N) \qquad (5)$$

where $\zeta_j \in (z_j-k, z_j+k)$, $k=\Delta z$.

And

$$\frac{\partial^2 E}{\partial t^2}(x_i,z_j,t_N)=\frac{E(x_i,z_j,t_N-l)-2E(x_i,z_j,t_N)+E(x_i,z_j,t_N+l)}{l^2} -\frac{l^2}{12}\frac{\partial^4 E}{\partial t^4}(x_i,z_j,\psi_N) \qquad (6)$$

where $\psi_N \in (t_N-l, t_N+l)$

Substituting Eqs. (3-6) into Eq. (2) we get,

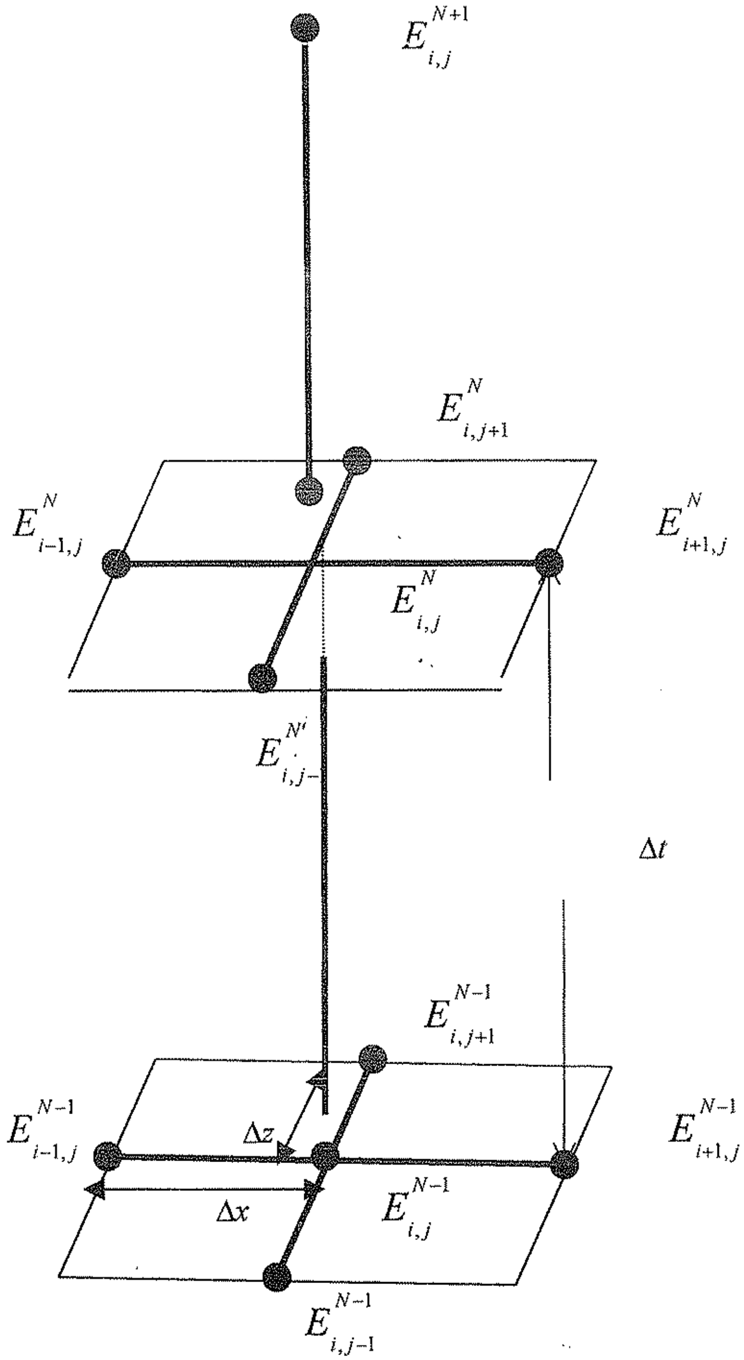


Fig. 1 Grid points for two space dimension

$$\mu_o \frac{\sigma E(x_i,z_j,t_N+l)-\sigma E(x_i,z_j,t_N)}{l}-\frac{l}{2}\frac{\partial^2 E}{\partial t^2}(x_i,z_j,\tau_N)$$
$$+\frac{\varepsilon_r E(x_i,z_j,t_N-l)-2\varepsilon_r E(x_i,z_j,t_N)+\varepsilon_r E(x_i,z_j,t_N+l)}{(cl)^2}-\frac{l^2}{12}\frac{\partial^4 E}{\partial t^4}(x_i,z_j,\psi_N)$$
$$=\frac{E(x_i-h,z_j,t_N)-2E(x_i,z_j,t_N)+E(x_i+h,z_j,t_N)}{h^2}-\frac{h^2}{12}\frac{\partial^4 E}{\partial z^4}(\xi_i,z_j,t_N)$$
$$+\frac{E(x_i,z_j-k,t_N)-2E(x_i,z_j,t_N)+E(x_i,z_j+k,t_N)}{k^2}-\frac{k^2}{12}\frac{\partial^4 E}{\partial z^4}(x_i,\zeta_j,t_N) \quad (7)$$

Assuming h, k and l are sufficiently small allowing us to ignore the local truncation error for each term in Eq. (7). Denoting the approximate value of E_y at (x_i,z_j,t_N) by $E_{I,J,N}$ (i.e. $E_{i,j,N}\approx E(x_i,z_j,t_N)$), the approximate value of E_y at (x_i,z_j,t_{N+1}) by $E_{I,J,N+1}$ and so forth, Eq. (7) reduced to

$$\mu_o \frac{\sigma E_{i,j,N+1}-\sigma E_{i,j,N}}{l}+\frac{\varepsilon_r E_{i,j,N-1}-2\varepsilon_r E_{i,j,N}+\varepsilon_r E_{i,j,N+1}}{(cl)^2}= \frac{E_{i-1,j,N}-2E_{i,j,N}+E_{i+1,j,N}}{h^2}+\frac{E_{i,j-1,N}-2E_{i,j,N}+E_{i,j+1,N}}{k^2} \quad (8)$$

Eq. (8) is then solved for the unknown field at time step N+1 at each node, The unknown field at time N+1 can be explicitly expressed in terms of linear or nonlinear permittivity ε_r in linear and nonlinear medium respectively, the conductivity σ and the known field value at time step N and N-1. So this method known as explicit method since the electric field is given explicitly, it has truncation error

$$\frac{l}{2}\frac{\partial^2 E}{\partial t^2}(x_i,z_j,\tau_N)+\frac{l^2}{12}\frac{\partial^4 E}{\partial t^4}(x_i,z_j,\psi_N)$$
$$-\frac{h^2}{12}\frac{\partial^4 E}{\partial x^4}(\xi_i,z_j,t_N)-\frac{k^2}{12}\frac{\partial^4 E}{\partial z^4}(x_i,\zeta_j,t_N)$$

Stability, consistence and convergent

In the non conducting electric media $(\sigma = 0)$, Eq.(8) becomes

$$\frac{\varepsilon_r E_{i,j,N-1} - 2\varepsilon_r E_{i,j,N} + \varepsilon_r E_{i,j,N+1}}{(cl)^2} = \frac{E_{i-1,j,N} - 2E_{i,j,N} + E_{i+1,j,N}}{h^2} + \frac{E_{i,j-1,N} - 2E_{i,j,N} + E_{i,j+1,N}}{k^2} \qquad (9)$$

Letting $\Delta x = \Delta z = h$, (*i.e.* $k = h$) and $r = \left(\frac{cl}{h\varepsilon_r^{1/2}}\right)^2 = \left(\frac{cl}{k\varepsilon_r^{1/2}}\right)^2$. Eq.(9) can then be rewritten as:

$$E_{i,j,N+1} + E_{i,j,N-1} = (2-4r)E_{i,j,N} + r(E_{i-1,j,N} + E_{i+1,j,N} + E_{i,j-1,N} + E_{i,j+1,N}) \qquad (10)$$

Let $e_{i,j,N+1} = E_{i,j,N+1} - \widetilde{E}_{i,j,N+1}$; where $E_{i,j,N+1}$ and $\widetilde{E}_{i,j,N+1}$ are the exact and the computational solution of Eq.(10) respectively. Since $E_{i,j,N+1}$ and $\widetilde{E}_{i,j,N+1}$ satisfy Eq. (10), we get

$$e_{i,j,N+1} + e_{i,j,N-1} = (2-4r)e_{i,j,N} + r(e_{i-1,j,N} + e_{i+1,j,N} + e_{i,j-1,N} + e_{i,j+1,N}) \qquad (11)$$

Assume that the error function $E(x,z,0)$ in x-z plane at $t = 0$ can be represented by Fourier series as [9]

$$E(x,z,0) = \sum_{i=0}^{N}\sum_{j=0}^{M} A_{i,j} e^{\sqrt{-1}\lambda_i x} e^{\sqrt{-1}\gamma_j z} \qquad (12)$$

where $A_{i,j}$ are the Fourier coefficients and can be neglected , λ_i, and γ_j are the frequencies, and *N*+1, *M*+1 are the mesh points in the *x-z* plane . Because of the linearity of the partial differential equation, we need to consider only the effect of terms $e^{\sqrt{-1}\lambda x}$ and $e^{\sqrt{-1}\gamma z}$ where λ and γ are constants, To investigate the propagation of these terms as t increases, we put

$$e(x_i, z_j, t_N) = e^{\sqrt{-1}\alpha x_i} e^{\sqrt{-1}\beta z_j} e^{\lambda t_N} = e_{i,j,N} \qquad (13)$$

Now we need to investigate how $e^{\lambda t_N}$ the amplification factor, will behave as we proceed to $t_N + 1$ or $N+1$. We need to determine whether the amplitude of the error will grow or die as we proceed to the next time level $N+1$.

Substitute Eq.(13) in Eq.(11) we get

$$
\begin{aligned}
& e^{\sqrt{-1}\alpha ih} e^{\sqrt{-1}\beta jh} e^{\lambda(N+1)l} + e^{\sqrt{-1}\alpha ih} e^{\sqrt{-1}\beta jh} e^{\lambda(N-1)l} \\
&= (2+4r) e^{\sqrt{-1}\alpha ih} e^{\sqrt{-1}\beta jh} e^{\lambda Nl} \\
&+ r\left(\begin{array}{l} e^{\sqrt{-1}\alpha(i-1)h} e^{\sqrt{-1}\beta jh} e^{\lambda Nl} + e^{\sqrt{-1}\alpha(i+1)h} e^{\sqrt{-1}\beta jh} e^{\lambda Nl} + \\ e^{\sqrt{-1}\alpha ih} e^{\sqrt{-1}(j-1)jh} e^{\lambda Nl} + e^{\sqrt{-1}\alpha ih} e^{\sqrt{-1}\beta(j+1)h} e^{\lambda Nl} \end{array} \right)
\end{aligned} \tag{14}
$$

Dividing each term of Eq. (14) by $e^{\sqrt{-1}\alpha ih} e^{\sqrt{-1}\beta jh} e^{\lambda Nl}$ we get

$$e^{\lambda l} + e^{-\lambda l} = (2-4r) + r\left(e^{-\sqrt{-1}\alpha h} + e^{\sqrt{-1}\alpha h} + e^{-\sqrt{-1}\beta h} + e^{\sqrt{-1}\beta h}\right) \tag{15}$$

Using $e^{\sqrt{-1}\lambda h} = \cos\lambda h + \sqrt{-1}\sin\lambda h$ in Eq. (15) we get

$$e^{\lambda l} + e^{-\lambda l} = (2-4r) + r(2\cos\alpha h + 2\cos\beta h) \tag{16}$$

Using $\cos\alpha h = 1 + 2\sin^2\dfrac{\alpha h}{2}$ in Eq. (16), yielding

$$e^{\lambda l} + e^{-\lambda h} = 2 - 4r\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right) \tag{17}$$

If we multiply both sides of Eq. (17) by $e^{\lambda l}$ we get

$$(e^{\lambda l})^2 - 2Ae^{\lambda l} + 1 = 0 \tag{18}$$

where

$$A = 1 - 2r\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right) \tag{19}$$

Hence the value of $e^{\lambda l}$ are

$$(e^{\lambda l})_1 = A + (A^2-1)^{\frac{1}{2}} \text{ and } (e^{\lambda l})_2 = A - (A^2-1)^{\frac{1}{2}}$$

As E does not increase exponentially with t and because the difference equation is a three time level approximation, a necessary condition for stability is that [12]

$$\left|e^{\lambda h}\right| \le 1$$

As r, α and β are real, $A \le 1$ by Eq. (19).

When $A < -1$, $\left|(e^{\lambda l})_2\right| > 1$ gives instability.

When $-1 \le A \le 1$, $A^2 \le 1$,

$$(e^{\lambda l})_1 = A + i(1 - A^2)^{\frac{1}{2}} \quad , \quad (e^{\lambda l})_2 = A - i(1 - A^2)^{\frac{1}{2}}$$

Hence $\left|(e^{\lambda l})_1\right| = \left|(e^{\lambda l})_2\right| = \left\{A^2 + \left(1 - A^2\right)\right\}^{\frac{1}{2}} = 1$,

showing that the necessary condition for stability is $-1 \le A \le 1$. By Eq. (19),

$$-1 \le 1 - 2r\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right) \le 1 \qquad (20)$$

The only useful inequality is

$$-1 \le 1 - 2r\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right) \qquad (21)$$

giving $r \le 1/\left(\sin\frac{\alpha h}{2} + \sin\frac{\beta h}{2}\right)$, then Eq.(20) is satisfied. Since α and β are arbitrary, $r \le \frac{1}{2}$. If $\Delta x \ne \Delta z$, *i.e.* $h \ne k$, then

$$-1 \le 1 - 2\left(r_x \sin^2\frac{\alpha h}{2} + r_z \sin^2\frac{\beta h}{2}\right) \le 1 \qquad (22)$$

where

$$r_x = \left(\frac{cl}{h\varepsilon_r^{1/2}}\right)^2 \text{ and } r_z = \left(\frac{cl}{k\varepsilon_r^{1/2}}\right)^2$$

The only useful inequality is

$$-1 \le 1 - 2\left(r_x \sin^2\frac{\alpha h}{2} + r_z \sin^2\frac{\beta h}{2}\right) \qquad (23)$$

Since α and β are arbitrary, then $(r_x + r_z) \le 1$,by substitution about r_x ,r_z in the last inequality we get the Courant-Fridrichs-Lewy (CFL) condition

$$\frac{(c\Delta t)^2}{\varepsilon_r}\left[\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta z)^2}\right]\le 1 \qquad (24)$$

where $\Delta x = h,\ \ \Delta z = k$ and $\Delta t = l$.

The stability condition in the nonlinear medium regions is given by [13]

$$(c\Delta t)^2\left[\frac{\partial}{\partial E_{i,j,N}}(\varepsilon_r E)_{i,j,N}\right]^{-1}\left[\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta z)^2}\right]\le 1 \qquad (25)$$

The term consistency means that the difference equation approximates the required partial differential equation and not other partial equation. Alternatively, we can say that the differential equation is a compatible with the required differential equation if the local truncation error of Eq. (1) goes to zero as $h,k,l \to 0$, no matter how this limit is taken. The real importance of the concept of consistency lies in a theory by Lax Equivalence Theorem [14], which states that *if a linear finite-difference equation is a consistence with a properly posed linear initial value problem then stability guarantees the convergence.*

The term convergent means that the difference between the exact solution and the exact finite-difference approximation as $h,k,l \to 0$.

Finally we conclude that Eq. (1) is a stable, consistence and convergent.

Results and comments

Figure (2) shows the structure of a slab waveguide [13]. A finite cross section is defined by enclosing the guide in a rectangular box where the side walls may be either electric or magnetic walls in order to include coupled structures. The structure consists from four sub-rectangular regions, three of them are linear with different relative permittivity, and the fourth regions is a nonlinear medium which the relative permittivity is a function of the electric field [15,16].

The structure is discretized by using grid sizes as $\Delta x = \Delta z = 0.025\mu m$ and time $\Delta t = 0.05 fs$. The CFL coefficient gives by the left hand side of Eq.(24) is 0.72, indicating that the scheme is stable, and in nonlinear media we take $\alpha = 25m^2 / v^2$.

Solving Eq0.(10) for the displacement at t = 0 (N = 0) , we get

$$E_{i,j,1} + E_{i,j,-1} = (2-4r)E_{i,j,0} + r(E_{i-1,j,0} + E_{i+1,j,0} + E_{i,j-1,0} + E_{i,j+1,0}) \quad (26)$$

The boundary condition at t = 0, in terms of central differences can be written as:

$$\frac{E_{i,j,1} - E_{i,j,-1}}{2\Delta t} = E_{i,j,0} \quad (27)$$

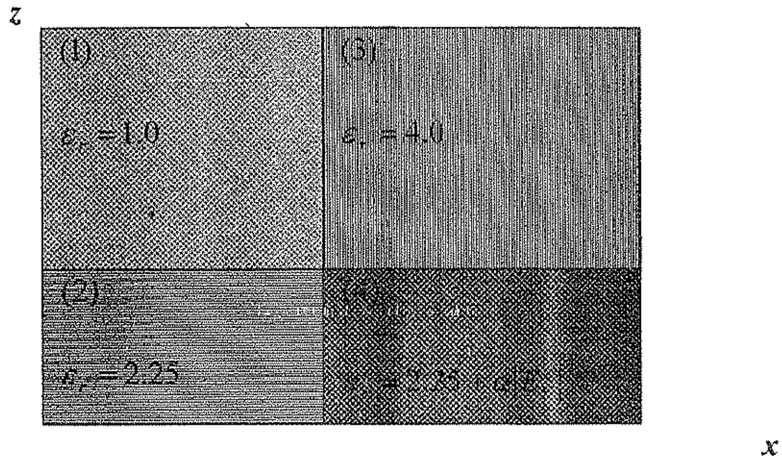


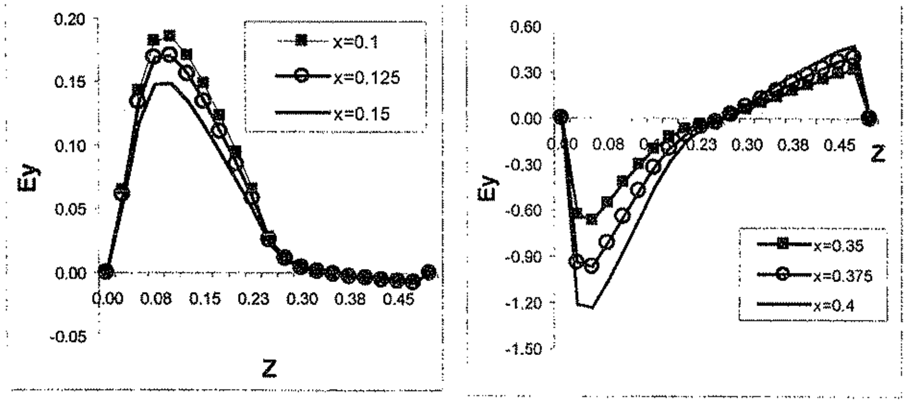
Fig. (2) 2-D slab waveguide Illustrate of the finite-difference method treatment.

Eliminating $E_{i,j,-1}$ between Eq. (26) and Eq.(27) gives

$$E_{i,j,1} = (1 + \Delta t - 2r)E_{i,j,0} + \frac{r}{2}(E_{i-1,j,0} + E_{i+1,j,0} + E_{i,j-1,0} + E_{i,j+1,0}) \quad (28)$$

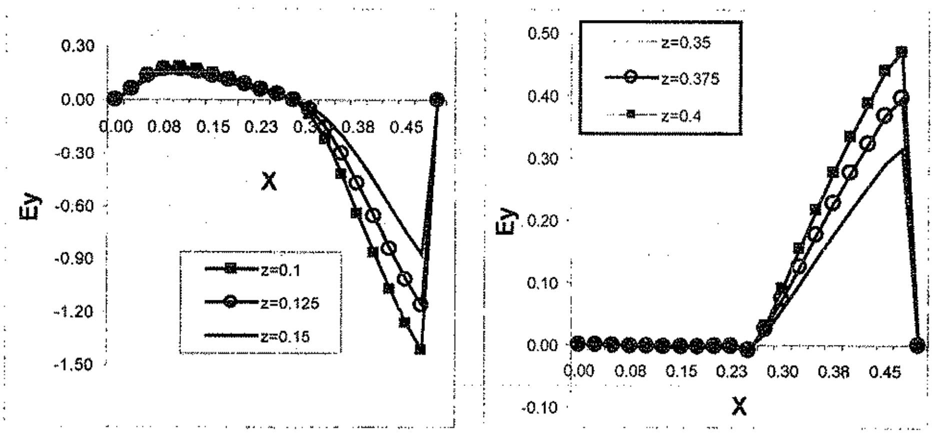
At $N = 1,2,\ldots$, we follow these equation to evaluate $E_{i,j,N+1}$

$$E_{i,j,N+1} = E_{i,j,N-1} + (2-4r)E_{i,j,N} + r(E_{i-1,j,N} + E_{i+1,j,N} + E_{i,j-1,N} + E_{i,j+1,N}) \quad (29)$$



(a) (b)

Fig. 3 Electric-field profile along the Z-direction at (a) x = 0.1, 0.125 μm and 0.15, (b) x = 0.35, 0.375 and 0.4 μm (T = 19 fs) obtained by using Explicit FDTD Method.



(a) (b)

Fig. 4 Electric-field profile along the x-direction at (a) z = 0.1, 0.125 and 0.15 μm, (b) z = 0.35, 0.375 and 0.4 μm (T = 19 fs) obtained by using Explicit FDTD Method.

For analysis and computation of the electric field profile distributions the FDTD approach described above has been adopted, and the prototype simulation FORTRAN program has been developed. Representative illustrations of numerical and computational results are displayed in Figs. (3-6). Here nonlinear waveguide structures are extracted from references [10,11].

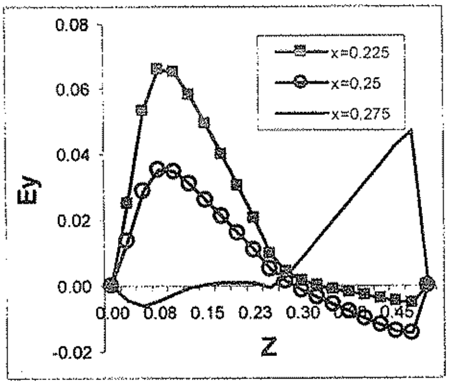


Fig. 5 Electric-field profile along the z- axis in linear and nonlinear regions at x = 0.225, 0.25 and 0.275 μm (T=19) obtained by the Explicit FDTD

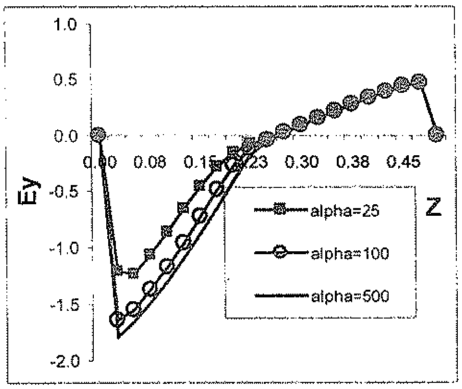


Fig. 6 Electric-filed profile along z-direction in nonlinear and linear region respectively at x = 0.4 μm and alpha α = 25, 100 and 500 m^2/V^2

Figures (3a) and (3b) show the electric field profile E_y along the z-direction at various value of points along the x-axis. Figures (4a) and (4b) shows the electric field profile E_y along the x-axis at various values of points along the z-axis. The effects of the nonlinear medium in both Figures can be seen easily. Both figures (3) and (4) show in clear some kind of an asymmetric picture and can be shifted a symmetric picture for some values at x and z dimensions, Figure (5) illustrates the electric field profile in the interface of two media in Figure (2). Figure (6) illustrates the effects of the nonlinearity on the field profile of E_y We note that the simulated field distributions reacts very sensitivity to the change in the waveguide structure as the nonlinear coefficient. The field profile E_y has a maximum value in the nonlinear regions and becomes low in the linear media.

Conclusion

A simple explicit Finite-Difference Time-Domain method is developed through the Taylor series expansion of the scalar wave equation. A Fortran program was written to calculate the electric-field profile by using Explicit FDTD Method in a linear and nonlinear rectangular structure. The stability condition, consistency and the convergency are demonstrated through a four layer wave guide structure and we found it is a very efficient and accurate by comparison with other results [15]. The described algorithms and approach also improve the traditional classical procedures and open new applications on photonic waveguide devices.

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