Peristaltic Motion of a Newtonian Fluid Through a Porous Medium

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ملخص البحث

تم في هذا البحث دراسة الحركة التمعجية لمائع نيوتوني غير قابل للإنضغاط خلال وسط مسامي في قناة منتظمة ذات بعدين متولد على جدارها موجة. تم كذلك تكوين وتحليل المشكلة باستخدام مفكوك اضطرابي بدلالة العدد الموجي كبارامتر.

يمكن أن يمثل هذا التحليل الوضع الباثولوجي لحركة البول في الحالب عندما تتكون بعض الحصوات في الجدار الداخلي للحالب. تم الحصول على صور تحليلية لمركبة السرعة المحورية ولتدرج الضغط، كما أنه تم حساب ضغط الامتلاء وقوة الاحتكاك عددياً. تبين من ذلك أن ضغط الامتلاء يزداد بنقصان معامل النفاذية، كما لوحظ أن كلاً من ضغط الامتلاء وقوة الاحتكاك لا تعتمد على بارامتر معامل النفاذية عند قيمة محددة لمعدل الانسياب. تم أيضاً دراسة جميع النتائج لقيم مختلفة للبرامترات الفيزيائية موضع الاهتمام.

Abstract

In this paper, peristaltic motion of an incompressible Newtonian fluid through a porous medium is studied in a two-dimensional uniform channel with a sinusoidal wave on its wall. The problem is formulated and analyzed using a perturbation expansion in terms of a variant of the wave number as a parameter.

This analysis can model the pathological situation of urine movement in the ureter when some stones are formed in its lumen. An analytic forms for axial velocity component and pressure gradient have been obtained. Moreover, the pressure rise and the friction force are computed numerically. It has been shown that the pressure rise increases as the permeability decreases. Further, it is noted that both pressure rise and friction force don't depend on permeability parameter at a certain value of flow rate. The results are studied for various values of the physical parameters of interest.

KEYWORDS: *peristalsis, Newtonian fluid, porous medium*

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1. Introduction

Peristaltic pump is a device for pumping fluids by means of a contraction wave travelling along a tube-like structure. The analysis of the mechanism responsible for peristaltic transport has been studied by many authors. Latham's investigation [10] seems to be the first study in this respect, and since that time several theoretical and experimental investigations have been made to understand peristaltic action in both mechanical and physiological situations. Some of these studies were made by Burns and Parkes [4], Barton and Raynor [2], Shapiro *et al.*[16], Roos and Lykoudis [15], Shukla *et al.* [17], Srivastava *et al.* [21,22,23], Srivastava and Srivastava [19,20], Srivastava [18], Elshehawey and Mekheimer [8], Bohme and Friedrich [3], El Misery *et al.* [6], Elshehawey *et al.*[7], Mekheimer *et al.* [11] and Elshehawey and Sobh [9].

Fluid motion through a porous medium has been studied by many authors. The usual starting point for the solution of such problems is Darcy's experimental law. Some studies about this point have been made by Varshney [24], Raptis *et al.* [12,14], Raptis and Peridikis [13], and El-Dabe and El-Mohandis [5].

With the above introduction in mind we aim to study the effect of porous medium on peristaltic motion of a Newtonian fluid. This model may be applied to the movement of the urine in the ureter in the presence of some stones in its lumen. Because of the complexity of the governing equations, a perturbation expansion with wave number as a parameter is obtained to a first order. The velocity field and pressure gradient are obtained in explicit forms. Moreover, the pressure rise per unit wavelength and the friction force are computed numerically and are plotted with the variation of the flow rate.

2. Formulation and Analysis

We shall consider a two-dimensional channel of uniform thickness 2a, filled with an incompressible Newtonian fluid through a porous medium occupying a semi-infinite region of the space. The walls of the channel are flexible and non-conducting, on which are imposed travelling sinusoidal waves of moderate amplitude. The geometry of the wall surface is

$$\overline{H}(\overline{X},\bar{t}) = a + b\sin\frac{2\pi}{\lambda}(\overline{X} - c\bar{t}), \tag{2.1}$$

where b is the wave amplitude, λ is the wave length, c is the propagation velocity, $\bar{t}$ is the time and $\overline{X}$ is in the same direction of the wave propagation.

Choosing moving coordinates $(\bar{x},\bar{y})$, (wave frame), which travel in the $\overline{X}$-direction with the same speed as the wave, the unsteady flow in the laboratory frame $(\overline{X},\overline{Y})$ can be treated as steady [16]. The coordinates frame are related through

$$\bar{x} = \overline{X} - c\bar{t}, \qquad \bar{y} = \overline{Y}, \tag{2.2}$$

$$\bar{u} = \overline{U} - c, \qquad \bar{v} = \overline{V}, \tag{2.3}$$

where $(\overline{U},\overline{V})$ and $(\bar{u},\bar{v})$ are the velocity components in the corresponding coordinate system.

Equation of continuity, equations of motion and the boundary conditions, respectively, take the following forms [1,16]

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{2.4}$$

$$\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial \bar{x}} - \frac{\mu}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \frac{\nu_1}{\bar{k}}\bar{u}, \tag{2.5}$$

$$\left(\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right)=-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{y}}-\frac{\mu}{\rho}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right)-\frac{\nu_1}{\bar{k}}\bar{v}, \tag{2.6}$$

$$\frac{\partial \bar{u}}{\partial \bar{y}}=0, \quad \bar{v}=0, \qquad \text{for} \quad \bar{y}=0, \tag{2.7a}$$

$$\bar{u}=-c, \quad \bar{v}=-c\frac{d\bar{H}}{d\bar{x}}, \qquad \text{for} \quad \bar{y}=\bar{H}. \tag{2.7b}$$

where $\bar{p}$ is the pressure, μ is viscosity, ρ is the density, ν_1 is the kinematic viscosity and $\bar{k}$ is the permeability parameter.

It is convenient to introduce the following dimensionless variables and parameters

$$x=\frac{\bar{x}}{\lambda}, X=\frac{\bar{X}}{\lambda}, y=\frac{\bar{y}}{a}, Y=\frac{\bar{Y}}{a}, t=\frac{c\bar{t}}{\lambda}, p=\frac{a^2\bar{p}}{c\lambda\mu}, v=\frac{\lambda\bar{v}}{ac}, u=\frac{\bar{u}}{c},$$
$$U=\frac{\bar{U}}{c}, \delta=\frac{a}{\lambda}, Re=\frac{\rho ca}{\mu}, k=\frac{\bar{k}}{a^2},$$

and $$H=\frac{\bar{H}}{a}=1+\frac{b}{a}\sin 2\pi x=1+\varphi\sin 2\pi x, \tag{2.8}$$

where

δ is the wave number,

Re is the Reynolds number,

$\varphi=\frac{b}{a}<1$, is the amplitude ratio.

Making use of (2.8) with (2.4-2.7), gives

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.9}$$

$$Re\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{u}{k}, \tag{2.10}$$

$$Re\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\delta^2\frac{v}{k}, \tag{2.11}$$

$$\frac{\partial u}{\partial y}=0, \quad v=0, \quad \text{for} \quad y=0, \tag{2.12a}$$

$$u=-1, \quad v=-\frac{dH}{dx}, \quad \text{for} \quad y=H. \tag{2.12b}$$

Eliminating the pressure between equations (2.10) and (2.11), we get

$$Re\,\delta\left[v\frac{\partial^2 u}{\partial y^2}-u\frac{\partial^2 v}{\partial y^2}-\delta^2\left(u\frac{\partial^2 v}{\partial x^2}-v\frac{\partial^2 u}{\partial x^2}\right)\right]=\frac{\partial^3 u}{\partial y^3}+\delta^2\left(\frac{\partial^3 u}{\partial y\partial x^2}-\frac{\partial^3 v}{\partial x\partial y^2}\right)$$

$$-\delta^4\frac{\partial^3 v}{\partial x^3}-\frac{1}{k}\frac{\partial u}{\partial y}+\frac{\delta^2}{k}\frac{\partial v}{\partial x}. \tag{2.13}$$

3. Rate of Volume Flow

The instantaneous volume flow rate in the fixed frame is given by

$$Q=\int_0^{\overline{H}}\overline{U}(\overline{X},\overline{Y},\overline{t})\,d\overline{Y}, \tag{3.1}$$

where $\overline{H}$ is a function of $\overline{X}$ and $\overline{t}$.

The rate of volume flow in the moving frame (wave frame) is given by

$$\overline{q}=\int_0^{\overline{H}}\overline{u}(\overline{x},\overline{y})\,d\overline{y}, \tag{3.2}$$

where $\overline{H}$ is a function of $\overline{x}$.

Using equation (3.2), one finds that the two rates of volume flow are related by

$$Q=\overline{q}+c\overline{H}. \tag{3.3}$$

The time-mean flow over a period $T = \dfrac{\lambda}{c}$ at a fixed position $\bar{X}$ is defined as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, d\bar{t}\,, \tag{3.4}$$

which may be written, using (2.1) and (3.3), as

$$\bar{Q} = \bar{q} + ac\,. \tag{3.5}$$

Defining the dimensionless time-mean flows Θ and F in the fixed and wave frame respectively as

$$\Theta = \frac{\bar{Q}}{ac} \qquad \text{and} \qquad F = \frac{\bar{q}}{ac}\,, \tag{3.6}$$

and making use of (3.6), then equation (3.5) may be rewritten as

$$\Theta = F + 1\,, \tag{3.7}$$

where

$$F = \int_0^{H(x)} u\, dy\,. \tag{3.8}$$

4. Perturbation Solution

If we expand the following quantities in a power series of the small parameter δ as

$$u = u_0 + \delta u_1 + O(\delta^2)$$

$$v = v_0 + \delta v_1 + O(\delta^2)$$

$$\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \delta\frac{\partial p_1}{\partial x} + O(\delta^2)$$

$$F = F_0 + \delta F_1 + O(\delta^2)\,, \tag{4.1}$$

Then the use of these expansions, with equations (2.9), (2.10), (2.12) and (2.13) will give the following systems

(i) System of order zero

$$\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}=0\ , \tag{4.2}$$

$$\frac{\partial p_0}{\partial x}=\frac{\partial^2 u_0}{\partial y^2}-\frac{1}{k}u_0, \tag{4.3}$$

$$\frac{\partial^3 u_0}{\partial y^3}-\frac{1}{k}\frac{\partial u_0}{\partial y}=0, \tag{4.4}$$

with the boundary conditions

$$\frac{\partial u_0}{\partial y}=0, \qquad v_0=0, \qquad \text{for} \quad y=0, \tag{4.5a}$$

$$u_0=-1, \qquad v_0=-\frac{dH}{dx}, \qquad \text{for} \quad y=H. \tag{4.5b}$$

The solution of this system for u_0 subject to the boundary conditions is

$$u_0=k\left(\frac{dp_0}{dx}\right)\left[\cosh\left(\frac{y}{\sqrt{k}}\right)\Big/\cosh\left(\frac{H}{\sqrt{k}}\right)-1\right]-\cosh\left(\frac{y}{\sqrt{k}}\right)\Big/\cosh\left(\frac{H}{\sqrt{k}}\right). \tag{4.6}$$

It is clear that as k tends to infinity , the velocity u_0 becomes

$$u_0=\frac{1}{2}\left(\frac{dp_0}{dx}\right)\left[y^2-H^2\right]-1.$$

The instantaneous volume flow rate F_0 in the moving coordinates is given by

$$F_0=\int_0^H u_0\,dy$$
$$=k\left(\frac{dp_0}{dx}\right)\left[\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)-H\right]-\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right),$$

which implies that

$$\frac{dp_0}{dx} = \left[F_0 + \sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)\right] \Big/ \left[k^{3/2}\tanh\left(\frac{H}{\sqrt{k}}\right) - kH\right]. \tag{4.7}$$

Using (4.2), (4.6) and (4.7), we obtain the alternative form of u_0 and v_0 as

$$u_0 = c_0 + c_1\cosh\left(\frac{y}{\sqrt{k}}\right), \tag{4.8a}$$

$$v_0 = -c_0' - c_1'\sqrt{k}\sinh\left(\frac{y}{\sqrt{k}}\right), \tag{4.8b}$$

where

$$c_0 = -k(b_0F_0 + b_1),\ c_1 = (a_0F_0 + a_1)$$

$$a_0 = \left[\sqrt{k}\sinh\left(\frac{H}{\sqrt{k}}\right) - H\cosh\left(\frac{H}{\sqrt{k}}\right)\right]^{-1},\ b_0 = \frac{a_0}{k}\cosh\left(\frac{H}{\sqrt{k}}\right),$$

$$a_1 = \left[a_0\sqrt{k}\sinh\left(\frac{H}{\sqrt{k}}\right) - 1\right] \Big/ \cosh\left(\frac{H}{\sqrt{k}}\right).\ b_1 = \frac{a_0}{\sqrt{k}}\sinh\left(\frac{H}{\sqrt{k}}\right). \tag{4.9}$$

(ii) System of order one

Equating the coefficients of δ on both sides in (2.9), (2.10) and (2.13), we get

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0\ , \tag{4.10}$$

$$Re\left(u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right) = -\frac{\partial p_0}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} - \frac{1}{k}u_1, \tag{4.11}$$

$$\frac{\partial^3 u_1}{\partial y^3} - \frac{1}{k}\frac{\partial u_1}{\partial y} = \mathrm{Re}\frac{\partial}{\partial y}\left(u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right), \tag{4.12}$$

with the boundary conditions

$$\frac{\partial u_1}{\partial y}=0, \qquad v_1=0\,, \qquad \text{for} \qquad y=0\,, \tag{4.13a}$$

$$u_1=0, \qquad v_1=0\,, \qquad \text{for} \qquad y=H\,. \tag{4.13b}$$

Solving this system for u_1 we obtain

$$u_1=k\left(\frac{dp_1}{dx}\right)\left[\cosh\left(\frac{y}{\sqrt{k}}\right)\Big/\cosh\left(\frac{H}{\sqrt{k}}\right)-1\right]+\mathrm{Re}\left[-\frac{c_0'c_1'}{4}(y^2-H^2)\cosh\left(\frac{H}{\sqrt{k}}\right)\right.$$
$$+\frac{\sqrt{k}}{4}(2c_0c_1'+3c_0'c_1)\left\{y\sinh\left(\frac{H}{\sqrt{k}}\right)-H\tanh\left(\frac{H}{\sqrt{k}}\right)\cosh\left(\frac{H}{\sqrt{k}}\right)\right\}$$
$$\left.+(c_0c_0'+c_1'c_1)\left\{\cosh\left(\frac{y}{\sqrt{k}}\right)\Big/\cosh\left(\frac{H}{\sqrt{k}}\right)-k\right\}\right]. \tag{4.14}$$

The instantaneous volume flow rate F_1 is given by

$$F_1=\int_0^H u_1\,dy$$
$$=k\left(\frac{dp_1}{dx}\right)\Big/\left[a_0\cosh\left(\frac{H}{\sqrt{k_1}}\right)\right]+\mathrm{Re}\left[(c_0c_0'+c_1c_1')\left(\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)-kH\right)\right.$$
$$\left.-\frac{c_0c_1'k}{2a_0}-\frac{k}{4}(2c_0c_1'+3c_0'c_1)\left\{\frac{1}{a_0}+H\tanh\left(\frac{H}{\sqrt{k}}\right)\sinh\left(\frac{H}{\sqrt{k}}\right)\right\}\right].$$

On solving this equation for $\frac{dp_1}{dx}$, one finds

$$\frac{dp_1}{dx}=b_0F_1+\mathrm{Re}\left(b_4F_0^2+b_5F_0+b_6\right), \tag{4.15}$$

where

$$b_2=b_0\left[\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)-kH\right],$$

$$b_3=-\frac{kb_0}{4}\left\{\frac{1}{a_0}+H\tanh\left(\frac{H}{\sqrt{k}}\right)\sinh\left(\frac{H}{\sqrt{k}}\right)\right\},$$

$$b_4 = 2kb_0b_3a_0' - a_0b_2a_0' - k^2b_0b_2b_0' + 3ka_0b_3b_0' - \frac{k^2b_0'}{2},$$

$$b_5 = 2kb_1b_3a_0' - a_1b_2a_0' - a_0b_2a_1' + 2kb_0b_3a_1' - \frac{k^2a_1b_0'}{2a_0} - k^2b_1b_2b_0'$$

$$+ 3ka_1b_3b_0' - \frac{k^2b_1'}{2} - k^2b_0b_2b_1' + 3ka_0b_3b_1',$$

$$b_6 = 2kb_0b_3a_1' - a_1b_2a_1' - k^2b_1b_2b_1' + 3ka_1b_3b_1' - \frac{k^2a_1b_1'}{2a_0}. \qquad (4.16)$$

Substitution of (4.15) into (4.14), give an alternative form for u_1 as

$$u_1 = c_2 + c_3\cosh\left(\frac{y}{\sqrt{k}}\right)$$

$$+ Re\left[c_4 + c_5\cosh\left(\frac{y}{\sqrt{k}}\right) + c_6y^2\cosh\left(\frac{y}{\sqrt{k}}\right) + c_7y\sinh\left(\frac{y}{\sqrt{k}}\right)\right], \qquad (4.17)$$

where

$$c_2 = -kb_0F_1\ , c_3 = a_0F_1,\ \ c_4 = -k\left(b_4F_0^2 + b_5F_0 + b_6 + c_0c_0' + c_1c_1'\right),$$

$$c_5 = \frac{\left(kb_4F_0^2 + kb_5F_0 + kb_6 + c_0c_0' + c_1c_1'\right)}{\cosh\left(\frac{H}{\sqrt{k}}\right)} + \frac{c_0'c_1H^2}{4}$$

$$-\frac{\sqrt{k}}{4}(2c_0c_1' + 3c_0'c_1)H\tanh\left(\frac{H}{\sqrt{k}}\right),$$

$$c_6 = -\frac{c_0'c_1}{4},\ \ c_7 = \frac{\sqrt{k}}{4}\left(2c_0c_1' + 3c_0'c_1\right). \qquad (4.18)$$

Noting that $F_0 = F - \delta\ F_1$, and for the first order approximation, one finds that the axial velocity component u and the pressure gradient $\frac{dp}{dx}$ can be expressed as

$$u = -k(b_0F + b_1) + (a_0 + a_1)\cosh\left(\frac{y}{\sqrt{k}}\right)$$

$$+\delta\, Re\left[a_2 + a_3\cosh\left(\frac{y}{\sqrt{k}}\right) + a_4 y^2 \cosh\left(\frac{y}{\sqrt{k}}\right) + a_5\, y\sinh\left(\frac{y}{\sqrt{k}}\right)\right], \quad (4.19)$$

$$\frac{dp}{dx} = b_0F + b_1 + \delta\, Re(b_4\, F^2 + b_5F + b_6), \quad (4.20)$$

where

$$a_2 = -k(b_4 + a_0a_0' + k^2b_0b_0')F^2 - k(b_5 + a_1a_0' + a_0a_1' + k^2b_1b_0' + k^2b_0b_0')F$$

$$-k(b_6 + a_1a_1' + k^2b_{01}b_1'),$$

$$a_3 = \frac{1}{4\cosh\left(\frac{H}{\sqrt{k}}\right)}\left[\left\{4kb_4 + 2k^{3/2}\, b_0a_0'H\sinh\left(\frac{H}{\sqrt{k}}\right) + 4k^2b_0b_0' + a_0(4a_0' + a_6b_0')\right\}F^2\right.$$

$$\left\{4kb_5 + k^{3/2}(2b_1a_0' + 2b_0a_1' + 3a_0b_1')H\sinh\left(\frac{H}{\sqrt{k}}\right) + 4a_0a_1' + a_1(4a_0' + a_6b_0')\right.$$

$$\left. + 4k^2(b_1b_0' + b_0b_1') - kb_1'a_0H^2\cosh\left(\frac{H}{\sqrt{k}}\right)\right\}F$$

$$\left. + \left\{4kb_6 + 4k^2b_1b_1' + a_1a_6b_1' + 2k^{3/2}b_1a_1'H\sinh\left(\frac{H}{\sqrt{k}}\right)\right\}\right],$$

$$a_4 = \frac{ka_0b_0'}{4}F^2 + \frac{k}{4}(a_1b_0' + a_0b_1')F + \frac{ka_1b_1'}{4},$$

$$a_5 = -\frac{k^{3/2}}{4}(2b_0a_0' + 3a_0b_0')F^2 - \frac{k^{3/2}}{4}(2b_1a_0' + 2b_0a_1' + 3a_1b_0' + 3a_0b_1')F$$

$$-\frac{k^{3/2}}{4}(2b_1a_1' + 3a_1b_1'),$$

$$a_6 = kH\left[3\sqrt{k}\sinh\left(\frac{H}{\sqrt{k}}\right) - H\cosh\left(\frac{H}{\sqrt{k}}\right)\right]. \quad (4.21)$$

The pressure rise per wavelength Δp_λ and friction force F_λ are given by

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx} dx, \tag{4.22}$$

and
$$F_\lambda = \int_0^1 H\left(-\frac{dp}{dx}\right) dx. \tag{4.23}$$

Substituting (4.20) into (4.22) and (4.23), and after evaluating the resulted integrals numerically we obtain both pressure rise and friction force. The results obtained are illustrated in Figs.1-4.

5. Results and Discussion

It is clear that our results are calculating the velocity and the pressure gradient in explicit form to the first order of δ without restrictions on the amplitude ratio and the Reynolds number. These results are to be considered as an extension of the work of Shapiro *et al.* [16], in which they used two separating expansions, the first is an expansion in powers of δ^2 with Re=0, while the second is an expansion in powers of Re^2 with δ=0. Further, our results are including the effect of the permeability parameter k.

To discuss the results obtained after substituting $\frac{dp}{dx}$ into equations (4.22) and (4.23) quantitatively, the integrals appear in these equations are evaluated numerically and are plotted in Figs.1-4.

The relation between pressure rise (Δp_λ) and flow rate (Θ) is displayed in Fig. (1) at Re=10, δ=0.02 and $\varphi = 0.2$ with different values of k. As shown, the pressure rise does not depend on k at a certain value of flow rate. It is important to note that the pressure rise increases as the permeability decreases. This is because of the resistance caused by the porous medium. In the case of ureter stones this causes renal colic (ureteric colic).

Fig.(2) shows the relation between pressure rise (Δp_λ) and flow rate (Θ) at Re=10, δ=0.02 and $\varphi = 0.8$ with different values of k. The behavior of the pressure rise is similar to its behavior in the first case, when $\varphi = 0.2$ and except for that, the values of the pressure rise are greater in this case.

The relation between friction force and flow rate given by equation (4.23) is plotted in Fig.(3) and Fig.(4) at Re=10, δ=0.02, (k=0.05, 0.1, 0.5, 1), for $\varphi = 0.2$ and $\varphi = 0.8$ respectively. We notice from these Figs. that the friction force increases as the flow rate increases. Also, it is to be noted that the friction force increases as the permeability decreases.

As a result, explicit forms for the axial velocity component u and the pressure gradient $\frac{dp}{dx}$ for peristaltic motion of a Newtonian fluid in a channel can be obtained from equations (4.19) and (4.20) by letting the permeability parameter k tends to infinity.

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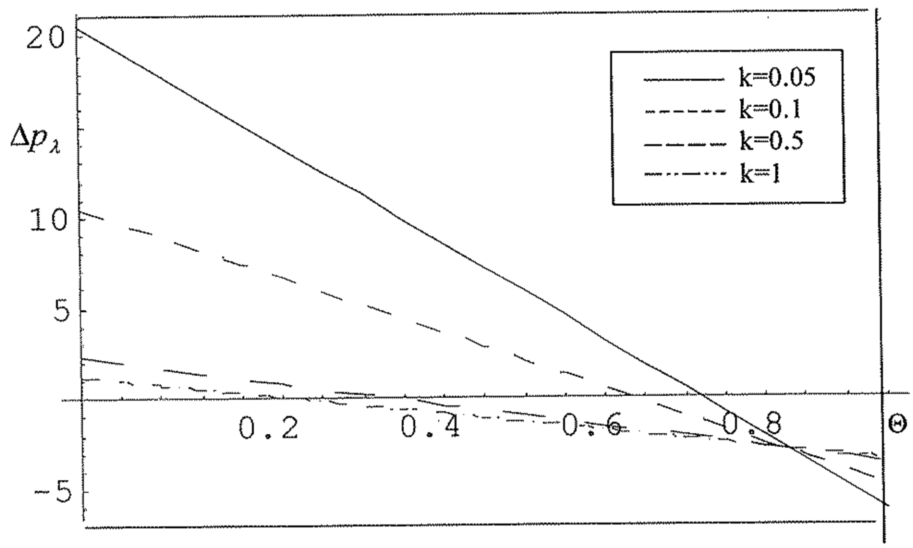

Fig.(1) Pressure rise versus flow rate at φ=0.2, δ=0.02, *Re*=10.

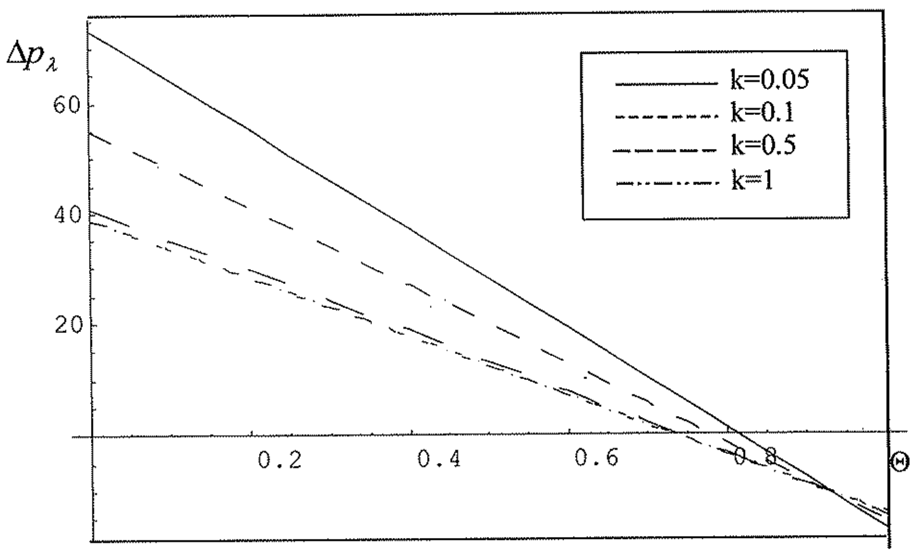

Fig.(2) Pressure rise versus flow rate at φ=0.8, δ=0.02, *Re*=10.

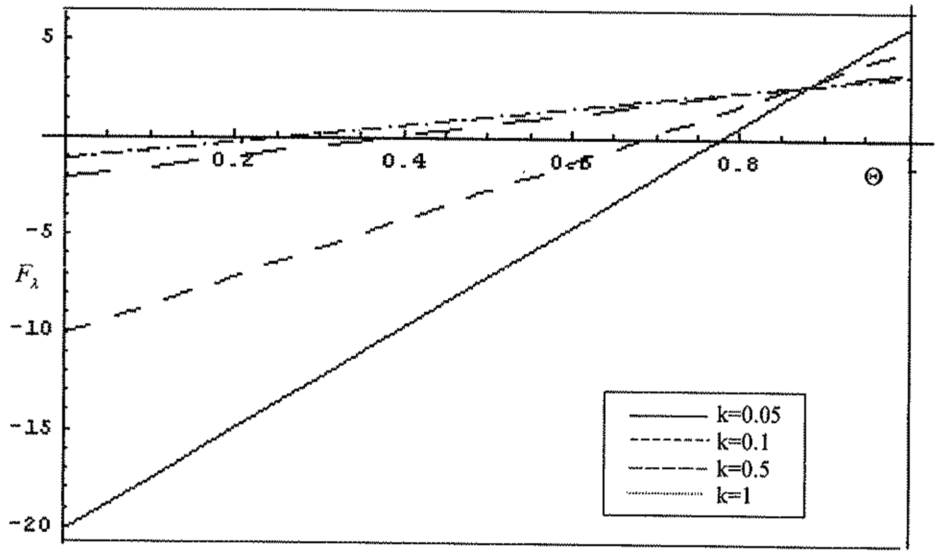

Fig.(3) Friction force versus flow rate at φ=0.2, δ=0.02, *Re*=10.

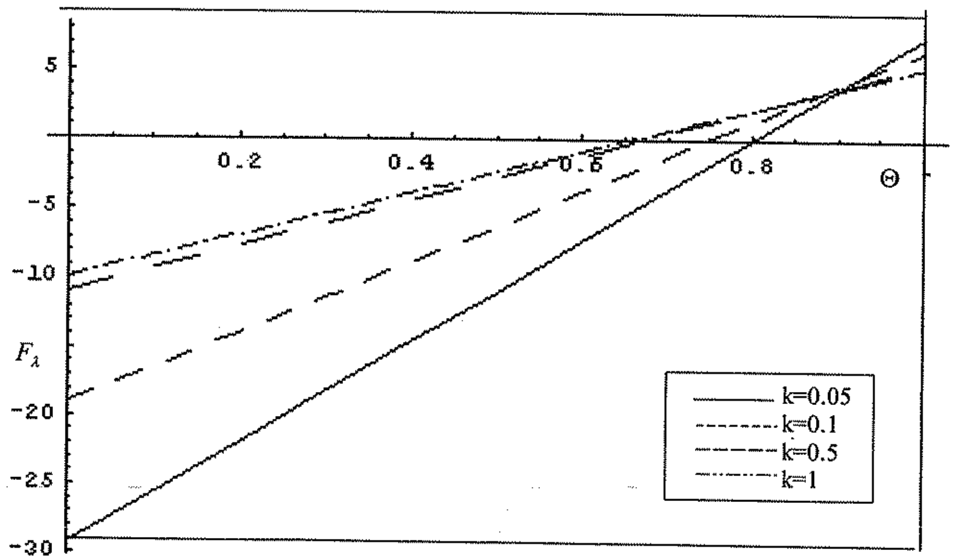

Fig.(4) Friction force versus flow rate at φ=0.8, δ=0.02, *Re*=10.