Analysis of Planar Scattering of Two Dimensional Structures Using The Finite-Difference Time-Domain Method

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الملخص

في هذا البحث قمنا بعمل خوارزمية لتحليل التشتت ثنائي الأبعاد يعتمد على طريقة الفروق المتناهية بالمجال الزمني، وقد استخدم نموذج Yee ذو المجال المفتوح ، ولحل هذه المشكلة يجب تحديد المجـــال واســـتخدمنا لذلـــك معادلـــة مـــيلر من الدرجة الأولى وشروط الامتصاص الحدية Mur absorbing boundary conditions . كمـــا تم تحديـــد الشروط التي تجعل هذه الطريقة مستقرة (Stability) ومتقاربة نحو الحلول المظبوطة .

Abstract

Algorithms based on Finite Difference Time Domain technique for the analysis of planar scatter structures are described. The scattering parameters for various geometrical structure are computed and the results found consistent with either proposed values or values previously published. We try to solve the scattering problem by using absorbing boundary condition (ABC) at boundaries. Excitation pulses that used are sinusoidal and Gaussian. An example illustrating the use Mur's first order boundary condition was introduced. The proposed approach could be used easily in designing various waveguide structures.

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1. Introduction

The Finite Difference Time Domain (FD-TD) technique is rapidly becoming one of the most widely used computational methods in electromagnetic problems [1-18]. There are several reasons for this, including the increased availability of low cost but powerful computers, and increasing interest in electromagnetic interactions with complicated geometries [2]. The combination of simplicity and power makes FD-TD such a popular method. The most satisfactory solution of a field problem is an exact mathematical one, but it is rare for electromagnetic (EM) problems to fall nearly into a class that can be solved by analytical methods. Complexity of the solution region, mixed types of boundary conditions, time-independent boundary condition and inhomogeneous medium are some reasons that make classical approaches fail. Whenever such complexity arises numerical solutions must be employed. The most powerful techniques of the numerical methods available for solving such problems is finite difference time domain method (FD-TD). The finite-difference-time domain (FD-TD) method is well-established numerical technique for the analysis of a great variety of electromagnetic problems. It is based on the direct discretization of Maxwell's time-dependent curl equations by using central finite-differences. The finite difference time domain has been gaining popularity because it has several advantages, for example, it leads to an explicit scheme (avoiding matrix inversion); the time domain solution is obtained directly. Simulation of electromagnetic problems using the finite difference time domain (FD-TD) method was first proposed by Kane Yee [1]. The method has been applied to various electromagnetic problems such as scattering, radiation and integrated-circuit component modeling [2-6]. Many of these applications involve modeling electromagnetic fields in an unbounded open space. Due to the limited storage space of computers, numerical computation have to be finite. Therefore, a certain type of boundary condition, which is called absorbing boundary condition (ABC) [7-14] or outer radiation boundary conditions (ORBC), need to be applied on outer boundaries of the computation domain to simulate the unbounded physical space. A popular and famous one is that suggested by Engquist and Majda [7] with the discretization given by Mur [8]. This is based on approximation of the outgoing wave equation by linear expressions using either a Taylor or a Pade approximation. Mur absorbing boundary conditions [8] which widely used are known nowadays as Mur absorbing boundary conditions. There are two forms of Mur absorbing boundary conditions known as Mur first order ABC and Mur second order ABC due to the degree of approximation used in both. Yee[1] hadn't used ABCs to model his unbounded scattering problem and instead he used a pure electric and magnetic reflectors at the boundaries. In this paper we try to solve the previous problem by using ABCs [7-14] at boundaries, and a brief discussion will be also introduced, as well as a modification on the structure will be made and treated.

2. FDTD method and Yee algorithm

The enormous difficulty of solving Maxwell equations is leading to the invention of various numeric methods of solution, among others the Yee algorithm [1]. One of the principal problems for the numerical method is the imposing of boundary conditions. We will see some boundary conditions that absorb an electromagnetic wave traveling outside the zone of interest and being able to simulate a planar waveguide extending to infinity. The FDTD method [8] provides solutions for time-dependent Maxwell equations. This method does not use potentials, but rather is based on

$\vec{E}$ and $\vec{H}$ fields in a three-dimensional space in such a way that all the $\vec{E}$ field components are surrounded by four circulating components of the field $\vec{H}$, and all the $\vec{H}$ field components are surrounded by four circulating components of the field $\vec{E}$ [8].

In an isotropic medium (μ, ε, and σ are independent of direction i.e. scalar), Maxwell's curl equations can be written as

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (2)$$

Every curl equation can be separated in the rectangular coordinates into the equivalent three scalar components. Both equations yield the following six equations. They are given (i.e. for Eqs. (1),(2) above) as follows:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \quad (3)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \quad (4)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \quad (5)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right) \quad (6)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \quad (7)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \quad (8)$$

where μ, ε and σ are the medium permitivity, permeability and conductivity respectively.

In order to determine discrete representations of these partial differential equations, the spatial region of interest is discreticized and the cartesian coordinates of the vector fields E and H are interleaved in both space time as specified by Yee [1]. By using the Yee's cell, the electric field components E_x, E_y and E_z are located on and assumed to be constant across each edge of the primary lattices cell with the magnetic field components H_x, H_y and H_z located on and assumed to be constant a cross each edge of the secondary lattice cell. In addition to a one-half spatial-cell displacement between *E* and *H*, there is also a one-half time-cell displacement.

We also assume that the media are piecewise uniform, i.e. $\Delta x = \Delta y = \Delta z = \Delta$ but $\delta t = \delta$; time increment is not. Following Yee's notation [1], we can define a grid point in the solution region as:

$$(i, j, k) = (i\Delta x, j\Delta y, k\Delta z, n\delta t) \text{ and}$$

$$f^n(i, j, k) = f(i\Delta x, j\Delta y, k\Delta z, n\delta t) \qquad (9)$$

Using the central difference approximations which second order accurate: we set,

$$\frac{\partial}{\partial x} f^n(i,j,k) = \frac{f^n(i+1/2, j, k) - f^n(i-1/2, j, k)}{\Delta} + O(\Delta^2) \qquad (10)$$

$$\frac{\partial}{\partial t} f^n(i,j,k) = \frac{f^{n+1/2}(i,j,k) - f^{n-1/2}(i,j,k)}{\delta} + O(\delta^2) \qquad (11)$$

Applying Eq.(10) and Eq.(11) to Maxwell's scalar equations above, we get the following equivalent finite difference approximate formulations, as

$$H_x^{n+1/2}(i,j+1/2,k+1/2)=H_x^{n-1/2}(i,j+1/2,k+1/2)+\frac{\delta t}{\mu(i,j+1/2,k+1/2)\Delta}\times\begin{pmatrix}E_y^n(i,j+1/2,k+1)-E_y^n(i,j+1/2,k)+\\E_z^n(i,j,k+1/2)-E_z^n(i,j+1,k+1/2)\end{pmatrix}\tag{12}$$

$$H_y^{n+1/2}(i+1/2,j,k+1/2)=H_y^{n-1/2}(i+1/2,j,k+1/2)+\frac{\delta t}{\mu(i+1/2,j,k+1/2)\Delta}\times\begin{pmatrix}E_z^n(i+1,j,k+1)-E_y^n(i,j,k+1/2)+\\E_x^n(i+1/2,j,k)-E_z^n(i+1/2,j,k+1)\end{pmatrix}\tag{13}$$

$$H_z^{n+1/2}(i+1/2,j+1/2,k)=H_z^{n-1/2}(i+1,j+1/2,k)+\frac{\delta t}{\mu(i+1,j+1/2,k)\Delta}\times\begin{pmatrix}E_x^n(i+1/2,j+1,k)-E_x^n(i+1/2,j,k)+\\E_y^n(i,j+1/2,k)-E_y^n(i+1,j+1/2,k)\end{pmatrix}\tag{14}$$

$$E_x^{n+1}(i+1/2,j,k)=\left[1-\frac{\sigma(i+1/2,j,k)\delta t}{\varepsilon(i+1/2,j,k)}\right]E_x^n(i+1/2,j,k)+\frac{\delta t}{\varepsilon(i+1/2,j,k)\Delta}\times\begin{pmatrix}H_z^{n+1/2}(i+1/2,j+1/2,k)-H_z^{n+1/2}(i+1/2,j-1/2,k)+\\H_y^{n+1/2}(i+1/2,j,k-1/2)-H_y^{n+1/2}(i+1/2,j,k+1/2)\end{pmatrix}\tag{15}$$

$$E_y^{n+1}(i,j+1/2,k)=\left[1-\frac{\sigma(i,j+1/2,k)\delta t}{\varepsilon(i,j+1/2,k)}\right]E_y^{n}(i,j+1/2,k)$$
$$+\frac{\delta t}{\varepsilon(i,j+1/2,k)\Delta}$$
$$\times\begin{pmatrix}H_x^{n+1/2}(i,j+1/2,k+1/2)-H_z^{n+1/2}(i,j+1/2,k-1/2)+\\ H_z^{n+1/2}(i-1/2,j+1/2,k)-H_z^{n+1/2}(i+1/2,j+1/2,k)\end{pmatrix} \quad (16)$$

$$E_z^{n+1}(i+1/2,j,k)=\left[1-\frac{\sigma(i,j,k+1/2)\delta t}{\varepsilon(i,j,k+1/2)}\right]E_z^{n}(i,j,k+1/2)$$
$$+\frac{\delta t}{\varepsilon(i,j,k+1/2)\Delta}$$
$$\times\begin{pmatrix}H_y^{n+1/2}(i+1/2,j,k+1/2)-H_y^{n+1/2}(i-1/2,j,k+1/2)+\\ H_x^{n+1/2}(i,j-1/2,k+1/2)-H_x^{n+1/2}(i,j+1/2,k+1/2)\end{pmatrix} \quad (17)$$

Yee [1] distributed the field components around a grid cell as shown in Fig. 1.
An additional consideration for the finite-difference solution to scatter structure is the stability of the numeric solution. The spatial increment Δ must be small compared to the wavelength (usually $\lambda\ /10$ where λ is the wavelength) or minimum dimension of scatterer. The condition on δ (time increment) which ensures stability of FD-TD mode is [16,17].

$$\delta \le \frac{1}{c_{\max}\sqrt{\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta y)^2}+\frac{1}{(\Delta z)^2}}} \quad (18)$$

where c_{max} is the maximum wave phase velocity within the model. Since $\Delta x = \Delta y = \Delta z = \Delta$, Eq.(18) can be expressed as:

$$\delta \leq \frac{\Delta}{c_{max}\sqrt{n}} \tag{19}$$

where n is the number of space dimensions.

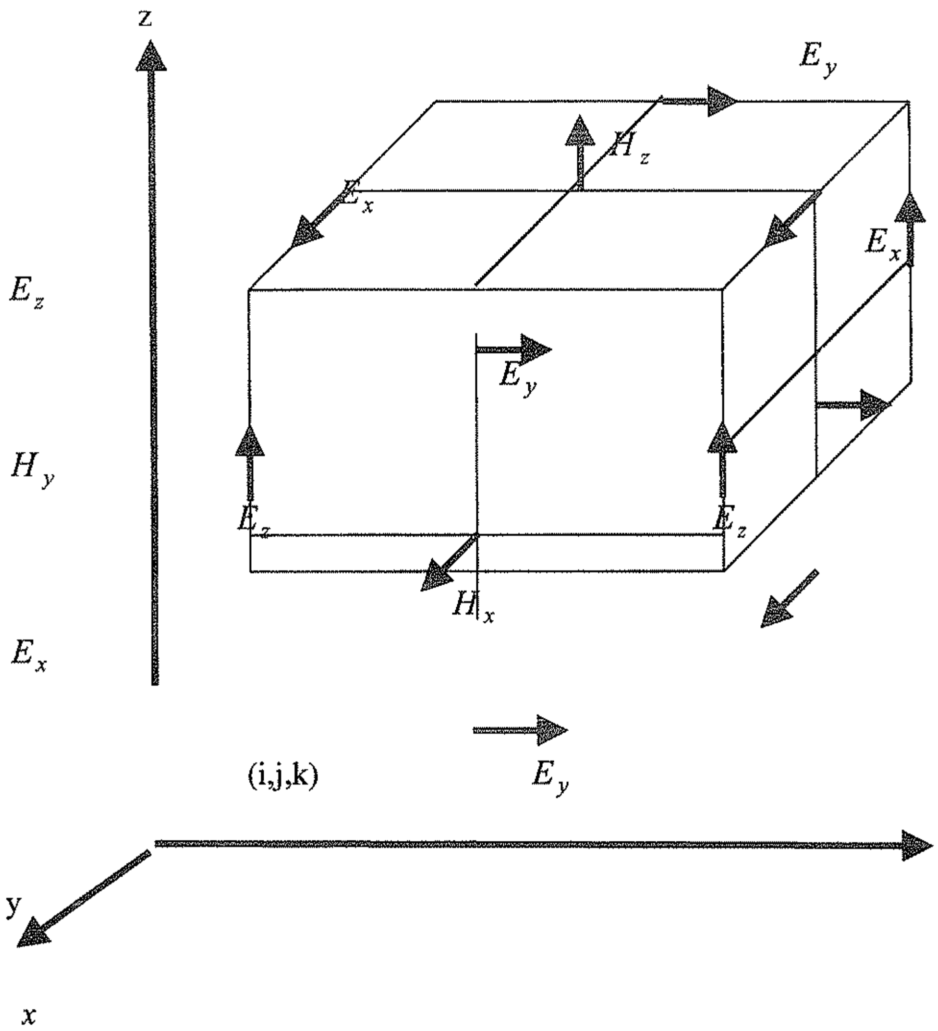


Fig. 1. Position of the electric and magnetic field vector components about a cubic cell of the Yee space lattice.

3. Source considerations

The two basic type of sources, which are plane waves and current line sources, are treated differently [17]. When the excitation source is assumed to be either a sinusoidal or Gaussian pulse plane wave, the computational domain is separated into an inner region, then the total fields are calculated, while in the outer region, only scattered fields are calculated. The scattered fields are defined to be the total field minus the incident plane wave. The incident fields are subtracted from the field quantities just within the inner regions, while they are used to calculate field quantities just beyond the inner region. The incident fields are added to the field quantities just outside the computation regions, while are used to calculate the field quantities just beyond the inner region. The incident fields are added to the field quantities just within the inner region. The implementation of the current line is straight forward. In this situation, the total fields are calculated within the enter computational domain. Current line sources are essentially point sources in the two-dimensional simulation. For the E-field polarization, an electric line source in z-direction is the source, while for H-filed polarization a magnetic line current in the z-direction is the source. Hence, the value of each current source is added to the value of either E_z or H_z, depending on the polarization, at a single point at every time step. For example the current source could be a sinusoidal or a Goussian pulse function.

4. Absorbing boundary conditions

Since the scatter structure is an open region problem, some method must be used to truncate and limit the size of spatial region. At the same time, this absorbing boundary condition (ABC) [7-14] must not produce extractions reflections back into the region of interest else it introduces errors in the solution. Various absorbing boundary conditions have been proposed for truncation of the FDTD mesh. Sheen, et. Al. [16] utilized a first order Mur absorbing boundary condition [8]which truncates the mesh. To understand the need for an ABC [7-14] in scattering and radiation problems, consider the field components are found at the boundaries. These cannot be updated using the usual FD-TD equations because some of the nearest-neighbor field components needed to evaluate the finite-difference curl enclosing it are outside the problem space and not available. The usual basis for ABCs [7-14]is to estimate the missing field components just outside the problem space by some means. These typically involves assuming that a locally plane wave is propagating out of the space, and estimating the fields for the outward traveling plane wave on the boundary by looking at the fields just within the boundary. Because in most situations the wave incident on the outer boundary will not be exactly plane, nor it will be normally incident. The absorbing boundary will not absorb the wave perfectly. There are many different schemes for accomplishing this. However, rather than provide the theoretical basis for ABC conditions, a popular and easily applied [8,17], or more particularly first or second Mur, depending on the order of approximating used to estimate the field on the boundary. A first order condition looks back one step in time and into space one cell location, a second order condition looks back two steps in time and inward two cell locations.

Consider that we are at the limit of x =0 of our FD-TD computational space. We decide that on this plane, we locate E_z and E_y fields components. Using these field components we can evaluate the finite difference curl equations in order to update the H_x magnetic field component at $x = 0$. So all nearest-neighbor field components will be available for updating the field components located at $x = \Delta x/2$ and beyond (at least to the maximum x dimension include in the structure space, at which location we must apply a ABC conditions) [7-14]. However, we can not update the E_z , E_y components at $x = 0$ with the usual FD-TD equations because the magnetic fields at $x = \Delta x/2$ are not available. We may update them using Mure expressions [17]. Let us consider the E_z component located at $x = 0$, $y = j\Delta y$, and $z = (k+1/2)\Delta z$, the first order Mur [8,17] estimates this field component as

$$E_z^{n+1}(0,j,k+1/2) = E_z^n(1,j,k+1/2) + \frac{c\Delta t - \Delta x}{c\Delta t + \Delta x}\left(E_z^{n+1}(1,j,k+1/2) - E_z^n(0,j,k+1/2)\right) \quad (20)$$

5. Numerical results

In the following we will treat the problem solved by Yee [1], with its own specifications, then we will apply the Mur's first order boundary conditions [16,17], In this example a rectangular conducting is imposed to a TM pulse propagating to the left along the x-axis, the pulse used is an upper part of a pure sinusoidal signal with a general mathematical expression given by [17]:

$$E_z(x,y,t) = \sin\left[\frac{(x-50a+ct)\pi}{8a}\right] \quad (21)$$

$$0 \le x - 50a + ct \le 8a$$

where a is constant, and c is the light speed in free space. Yee [1]had taken into account that the boundaries must be far enough from the scatter to notice the pulse behavior while going forward and backward from the scatterer. The interaction due to the boundaries close to the scatter is also avoided. At the boundaries, Yee [1]placed two kinds of conductors, a pure magnetic reflector (conductor) at $y = 0$, $y = y_{max}$ and a pure electric conductor at $x = 0$, $x = x_{max}$.

Yee[1] used the scatterer dimensions to be $4\alpha \times 4\alpha$ units and discretized the whole region into equally grids with $\Delta x = \Delta y = \alpha / 8$ and $c\Delta t = \alpha / 16$.

The pulse duration was α units. The whole geometry of the structure is shown in Fig.2.

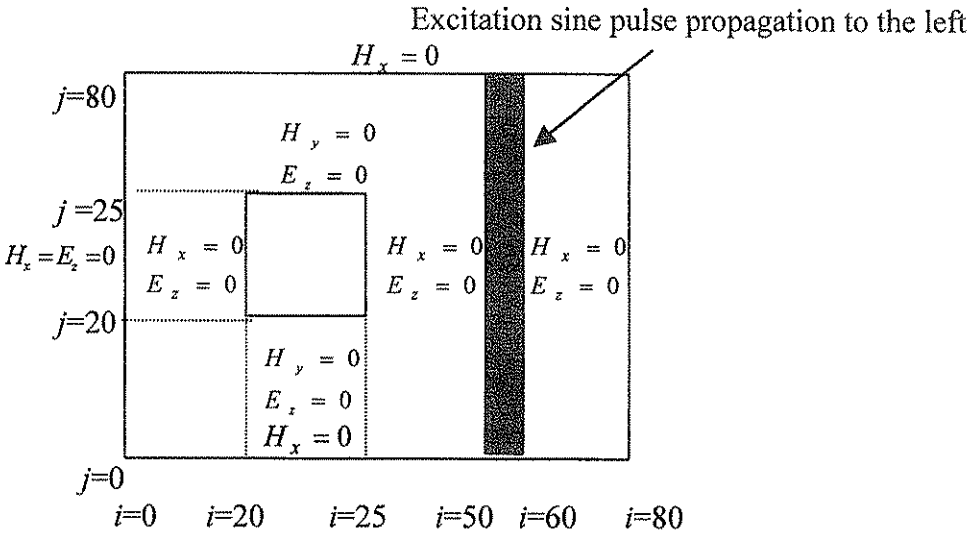


Fig. 2 Geometry and boundary conditions of Yee's problem.

Assuming the current density $\vec{j} = 0, E_x = E_y = 0$ and $\partial / \partial z = 0$, then Maxwell's curl equations for a TM_z wave are given as follows:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial y} \quad (22)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x} \quad (23)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \tag{24}$$

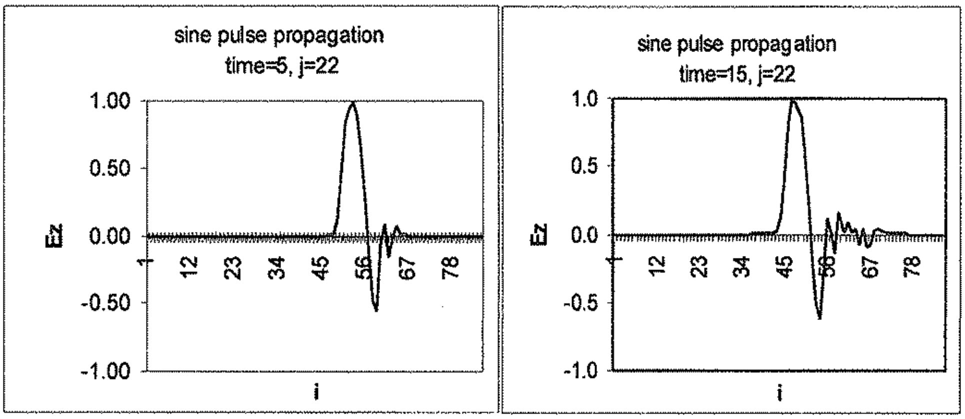
The approximate FD-TD formulation to the above equations are given by:

$$H_x^{n+1/2}(i,j+1/2,k+1/2) = H_x^{n-1/2}(i,j+1/2,k+1/2) + \frac{\delta t}{\mu\Delta}\left(E_z^n(i,j,k+1/2) - E_z^n(i,j+1,k+1/2)\right) \tag{25}$$

$$H_y^{n+1/2}(i+1/2,j,k+1/2) = H_Y^{n-1/2}(i+1/2,j,k+1/2) + \frac{\delta t}{\mu\Delta}\left(E_z^n(i+1,j,k+1/2) - E_z^n(i,j,k+1/2)\right) \tag{26}$$

$$E_z^{n+1}(i,j,k+1/2) = E_z^n(i,j,k+1/2) + \frac{\delta t}{\varepsilon\Delta}\begin{pmatrix} H_y^{n+1/2}(i+1/2,j,k+1/2) - H_y^{n+1/2}(i-1/2,j,k+1/2) + \\ H_x^{n+1/2}(i,j-1/2,k+1/2) - H_x^{n+1/2}(i,j+1/2,k+1/2) \end{pmatrix} \tag{27}$$

The results are shown down in Fig. 3, which is equivalent to those obtained by Yee [1].



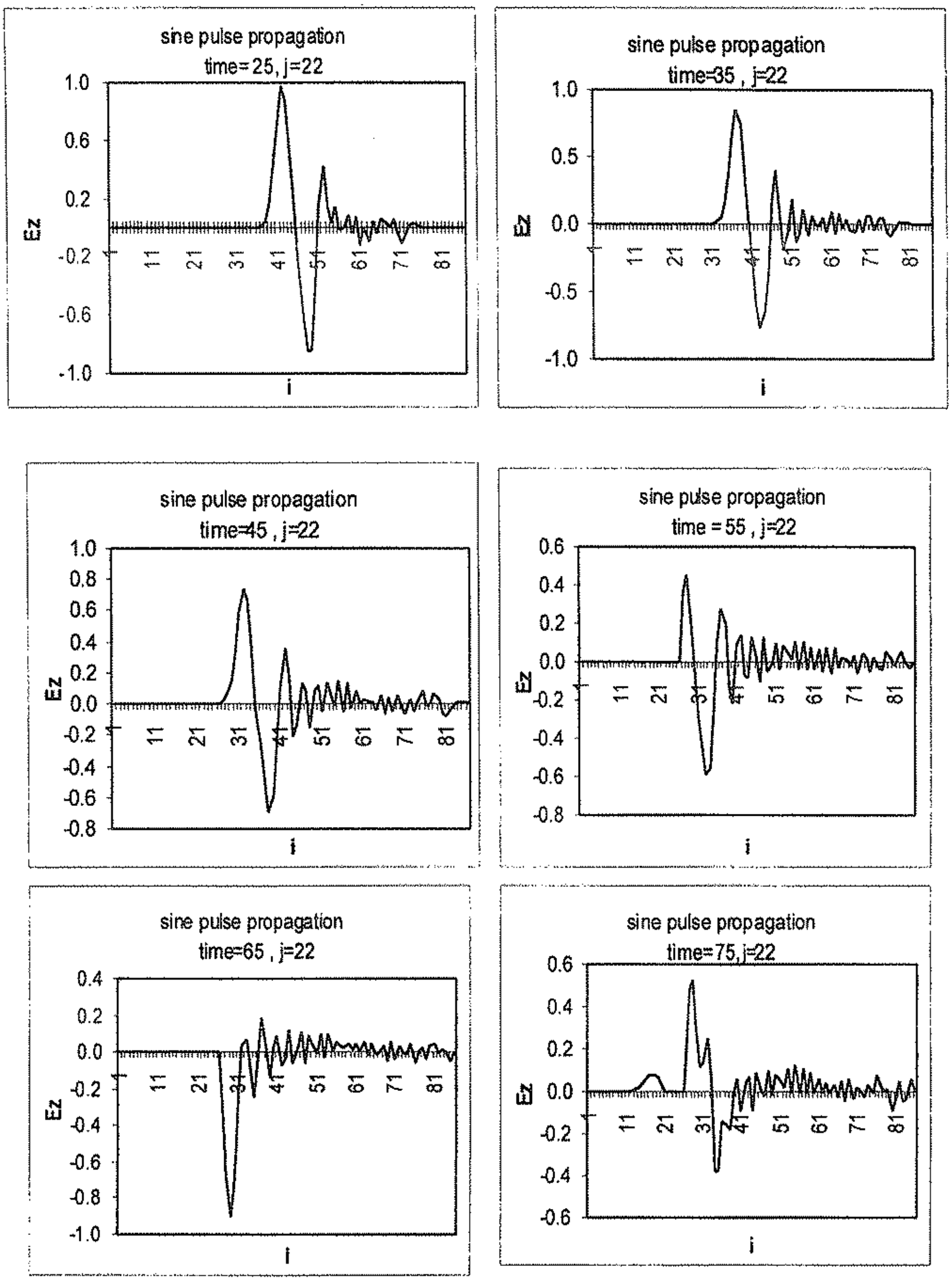


Fig. 3. E_z of the TM wave for various time steps, and *j*=22

In order to avoid this problem Mur's first order boundary condition will be applied as absorbing boundary. To show that Mur's first order boundary condition is a satisfactory and suitable, we let a Gaussian pulse[17], given mathematically as:

$$E_z(i,j,t) = \exp(-\gamma(i-\beta)^2) \tag{28}$$

where γ and β are constants, to propagate to the right along the x-axis in the solution region illustrated in Fig. 4 , where the absorbing boundary condition (ABC) is placed 1λ from a. scatter of size $8\Delta \times 8\Delta$, (where $\Delta = \lambda/20 = 3/20 = 0.15m$), behavior of the pulse at various time steps is obtained and shown in Fig. 5. All computer codes were written in Fortran.

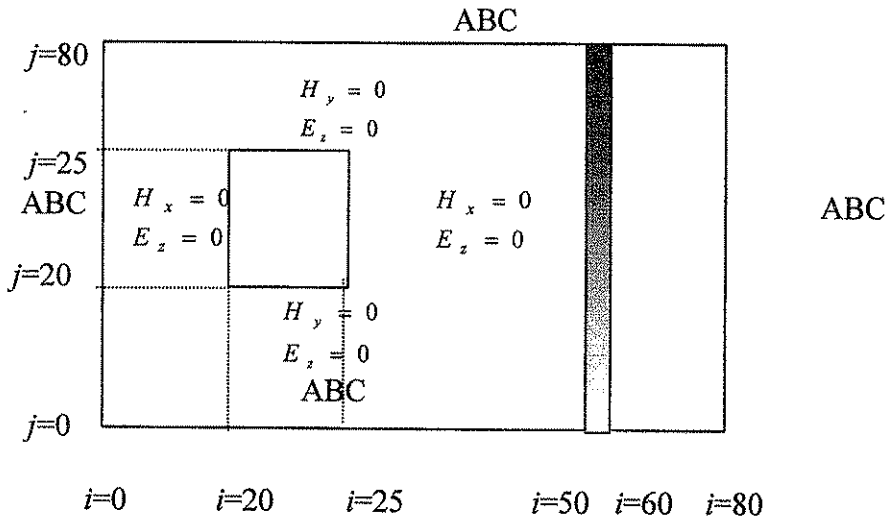
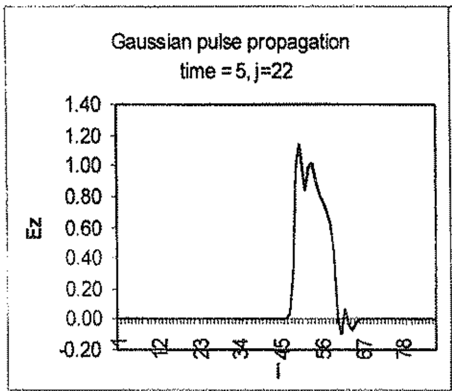
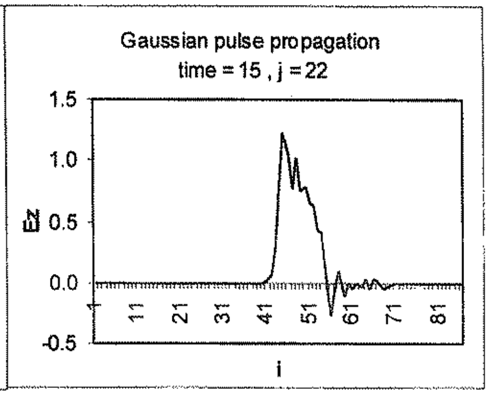
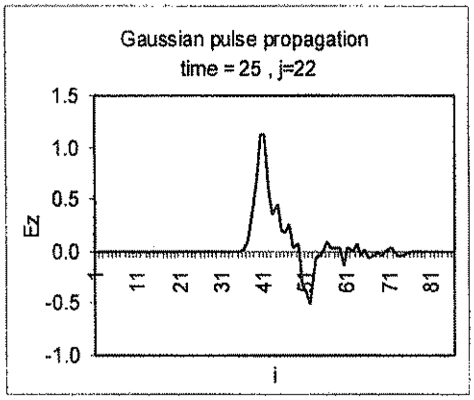
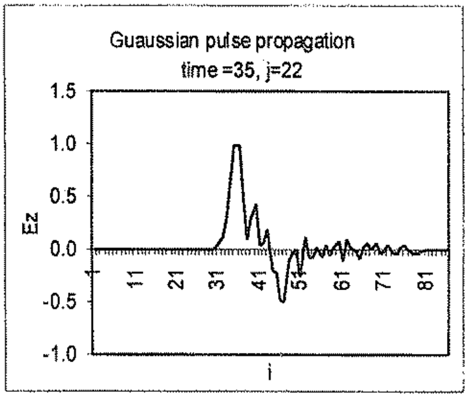


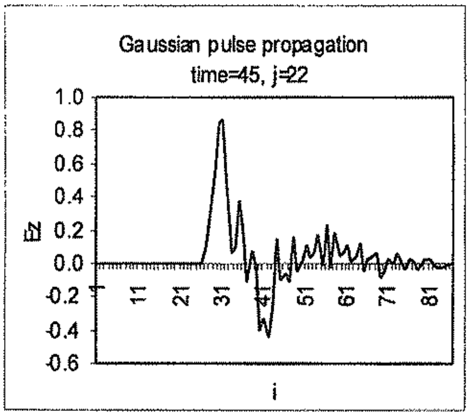
Fig. 4 Geometry of a scatterer with sides $8\Delta \times 8\Delta$ located into the solution region, where boundaries 1λ from scatterer

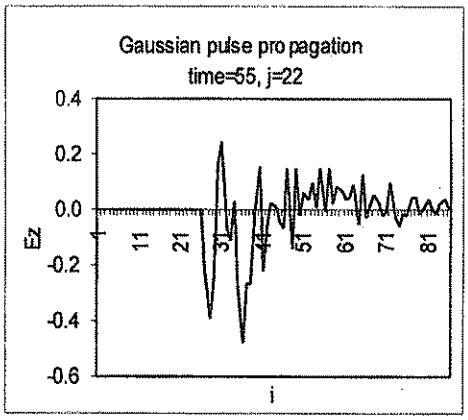

Gaussian pulse propagation
time = 5, j=22
Ez
i


Gaussian pulse propagation
time = 15 , j = 22
Ez
i


Gaussian pulse propagation
time = 25 , j=22
Ez
i


Guaussian pulse propagation
time =35, j=22
Ez
i


Gaussian pulse propagation
time=45, j=22
Ez
i


Gaussian pulse propagation
time=55, j=22
Ez
i

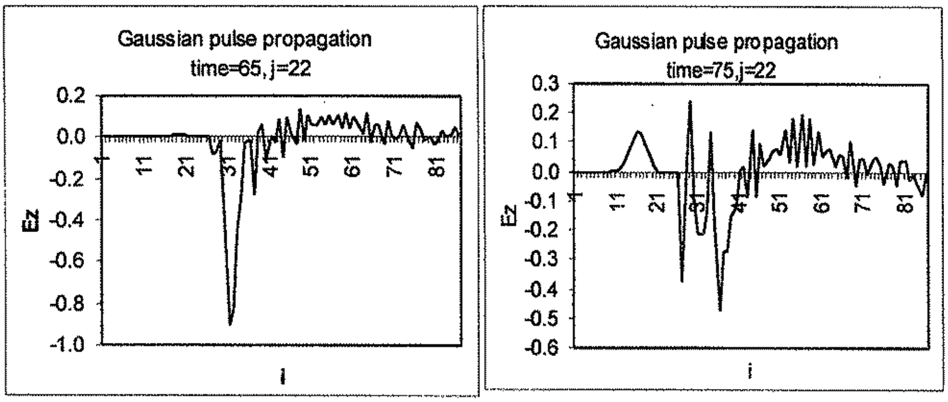


Fig. 5. E_z of the TM Gaussian pulse for various time steps, where ABC is far 1 λ from the scaterer

6. Conclusion

A complete review of the finite difference time domain(FD-TD) method was accomplished. An improved Finite Difference Time Domain approach for planar scatter structures has been presented. The requirements for stability are illustrated and applied. A computer program in FORTRAN was written and successfully implemented to obtain the results. A great similarity between our results and Yee's results. However, Yee did not take into account the infinite nature of the surrounding medium. An illustrated example is produced to show how to use Mur's first order boundary condition. In order to solve an open initial boundary value problem, an absorbing boundary condition has to be used. For the reliability and ease in representation, Mur's first order boundary condition was used. An example illustrating how to use Mur's first order boundary condition was introduced. The proposed approach could be used easily in designing some future waveguide structures.

1. K. S. Yee, Numerical solution of initial boundary value problems involving Maxwell,s equation in isortopic media, IEEE Trans. Antenna and propag Ap17 (1966), 302-307.

2. A.Taflove, Aplication of the finite-difference-time-domain method to sinsoidal steady-state electromagnetic penetration problems, IEEE Trans Electromagn Compat EMC-22 (1980), 191-202.

3 R. W. M. Lan and R. G. Sheppare, The Modeling of Biological systems in three dimension using the time domain finite-difference method, 2 parts Phys Med Biol 31 (1986), 1247-1266.

4 X. Young, A full finite difference time domain implementation for Radio wave propagation in a plasma, Radio Sci 29 (1994), 1513-1523.

5 W. K. Gwarek, Analysis of an arbitrary-shaped planer circuit-a time-domain approach, IEEE Trans Micro Tech MTT-33 (1985), 1067-1072.

6. F. Perez, et al., Calculation of the Field-intensity pattern in optical planar waveguide by the finite-difference time-domain method J Opt Commun 20 (1999), 202-209.

7. I. Engquist and A. Majda, Absorbing boundary conditions for numerical simulation of waves, Mathematics of computation 31 (1977), 629-651.

8. G. Mur, Absorbing boundary conditions for finite-difference Approximation of the time-domain electromagnetic-field equations, IEEE Trans. Electromagn. Compact EMC-23, (1981), 377-382.

9. A. C. Reynolds, Boundary conditions for the numerical solution of the wave propagation problems, J Phys 43 (1978), 1099-1110.

10. R. L. Higdon, Numerical absorbing boundary conditions for the wave equation, Math. Comp 49 (1987), 65-91.

11. R. W. Ziolkowski, The design of Maxwellian absorbers for numerical

boundary conditions and for practical applications using engineered artificial materials, IEEE Trans Antennas Propagat 45 (1997), 656-671.

12. O. m. Ramahi, Complementary boundary operators for wave propagation problems, J Comput Phys 133 (1997), 113-128.
13. A. Taflove , Computational electrodynamics:The finite-difference time-domain method, Artech house Inc, 1995.
14. I. Taflove and M. E. Brodwin, Numerical solution of steady state electromagnetic scattering problems using the time dependent Maxwell's equations, IEEE Trance. Microwave Theory Tech MTT-23 (1975), 623-630.
15. K. Kunz, R. J. Luebbers, The finite difference time domain method for electromagnetics, CRC P ress Inc Florida, 1993.
16. I. M. Sheen, S. M. Ali, M. D. Abouzahra and J. A. Kong, Application of the three-dimensional finite difference-time-domain method to the analysis of planer microstrip circuits, IEEE Trans. Microwave Theory Tech MTT-38 (1990), 849-857.
17. W. K. Gwarek, Analysis of arbitrary shaped two-dimensional microwave circuits by finite-difference time-domain method, IEEE Trans Micro Theo Tech 36 (1988), 738-744.
18. M. M. Shabat, M. A. Abdel-Naby, D. Jäger, and N. M. Barakat,, Numerical and analytical solutions of dispersion equation in lossy non-linear wave guiding system, Microwave and Optical Technology Letter 22 (1999), 273-278.