Nonlinear TE Surface Waves in Dielectric material sandwiched between LHM and Nonlinear Nonmagnetic LHM Structure

Dr. Ass'd I. Ass'ad *

الملخص

قمنا بتصميم مادة غير خطية غير مغناطيسية ذات معامل نفاذية كهربية (ثابت العزل) سـالبة وكذلك معامل انكسار سالب. قمنا بدراسة وتحليل مميزات وخصائص وتشتت انتشار الموجات المستعرضة الكهربية (TE) خلال طبقات متعددة وهي مادة غير خطية غير مغناطيـسية ذات ثابت العزل سالبة وخطية غير مغناطيسية ذات ثابت العزل سالبة ، بينهما مـادة خطيـة ذات ثابت العزل موجب. وتمت الدراسة باستخدام معادلات ماكسويل . تم تحليل ومناقشة كـل مـن ثابت الانتشار ومعامل التضعيف كدالة في التردد وسمك الطبقة الوسطي. كذلك تمـت دراسـة سلوك وانتشار الطاقة المصاحبة لهذه الموجات.

ABSTRACT

We introduce the design of nonlinear nonmagnetic left handed material. The dispersion relation has been derived for a slab waveguide of dielectric material sandwiched between linear and nonlinear nonmagnetic LHM slabs. A numerical Code was built to find the dispersion relation of the nonlinear transverse electric field (TE) surface waves in the structure and the power flow. Also, we study the effect of the nonlinearity, the slab thickness, and operating frequency on the effective wave index and power flow. We found that the wave effective refractive index and the power flow are dependent on the frequency, thickness and the nonlinearity of the structure.

Keywords: LHM, Nonlinear Nonmagnetic

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* Department of Physics, Faculty of Applied Sciences, Al-Aqsa University, Palestine, Email: assad@alaqsa.edu.ps.

INTRODUCTION:

A medium which have both the dielectric constant ε and the magnetic permeability μ assume negative values will have peculiar properties. This has been demonstrated theoretically by Veselago, who shown the propagation direction of an electromagnetic wave will be opposite to its energy flow direction in such material [1-3]. He used the term Left-Handed Material (LHM) for this class of material, and the wave vector **k,** figure 1-a, and the phase velocity, exhibit a sign opposite to that in a conventional right-handed (RH) material, figure 1-b,

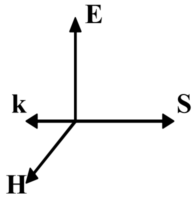

Figure (1-a) : plane wave in LH

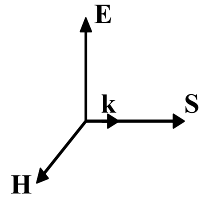

Figure (1-b) plane wave in RH

but the Poynting theorem is still given by $S = E \times H^*$, as in figure(1) which exhibit the triplet (**E**, **H**, **S**) in case of (LH and RH) [4].

The demonstrations of the left-handed material opened a wide door to design a novel devices based on the electromagnetic wave propagation in this class of materials [5,6]. Recently, Smith et al. found negative effective permittivity ε and permeability μ simultaneously in a system consisting of split ring resonators and metal wires [7], and Shelby et al. have observed negative values of the refraction index *n* in this system [8]. The first application of the negative-refraction material was suggested by Pendry [6], who demonstrated that a slab of a lossless negative-refraction material can provide a perfect image of a point source.

All properties of left-handed materials were studied in the linear regime of wave propagation when both ε and μ of material in some range frequency were assumed to be independent of the intensity of the electromagnetic field. However, any future effort in creating tunable structures where the field intensity changes the transmission properties of the composite structure would require the knowledge of the nonlinear properties of left-handed metamaterials for example of a lattice of split-ring resonators and wire with a nonlinear dielectric in general form $\varepsilon_D = \varepsilon_D\left(\left|E^2\right|\right)$. Recently Zharov et al [9]. have considered and analyzed, for the first time the nonlinear properties of left-handed metamaterials. They had considered two dimensional composite structures consisting of square lattice of the periodic arrays of conducting wires and split-ring resonators, and the cell size of the structure is much smaller than the wavelength of the propagation of the electromagnetic field. Importantly, the macroscopic electric field in such composite structures can become much higher than the macroscopic electric field carried by the propagating electromagnetic wave [10]. This provides a simple physical mechanism for enhancing nonlinear effects in the resonant structure with the left-handed properties, and the study of nonlinear left-handed material properties are expected to be quite unusual, and ignited a new gate for researchers in electromagnetic.

This paper is organized as follow; in section II, we have studied the theoretical derivation of the dispersion relation which governs the electromagnetic waves propagation in dielectric medium covered by LHM and nonlinear nonmagnetic LHM structure in GHz range of frequency. In section III, the dispersion relation has been solved to explore the characteristics of the structure. Section IV is purely devoted for the conclusion.

II. Theory :

Figure 2 shows the structure under investigation. we present the dispersion relation for TE wave propagation in the x-axis with propagation wave constant β in the form $e^{ik_0(\beta x - ct)}$, where $\beta = k / k_0$, k is the complex propagation constant, and k_0 is the free space wave number which equals ω/c, where c is the velocity of light, and ω is the applied angular frequency.

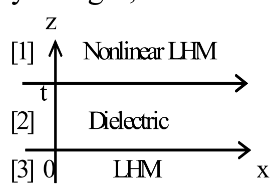

Fig. (2) TE surface waves waveguide composed of nonlinear nonmagnetic LHM

Consider a linear dielectric film (medium 2) of thickness (*t*), and dielectric constant ε_2. The linear dielectric film is bounded by linear LHM ε_{eff}^{L} (medium 3) in the region $z < 0$, and nonlinear nonmagnetic LHM cover (medium 1) in the region $z > t$, and its dielectric constant is ε_{eff}^{NL}. Here we consider the Kerr like nonlinearity of the dielectric in the composite material [9], i.e.

$$\varepsilon_{eff}^{NL} = \varepsilon_D\left(|E|^2\right) - \frac{\omega_p^2}{\omega^2} \quad (1)$$

Where $\varepsilon_D = \varepsilon_1 + \alpha E_y^2$, ε_1 is the linear part of the dielectric constant, and α is the nonlinear coefficient. The permeability of the nonlinear nonmagnetic left handed material is considered as $\mu = 1$ [11]. The electromagnetic field components are,

$$E = \left(0, E_y, 0\right)e^{ik_0(\beta x - ct)}, \quad (2)$$

$$H = \left(H_x, 0, H_z\right)e^{ik_0(\beta x - ct)}, \quad (3)$$

In nonlinear LHM cover ($z > t$):

Substitution of equations (2) and (3) into Maxwell's equation yields the following nonlinear differential equation to satisfy in the nonlinear nonmagnetic LHM slab:

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left(\beta^2 - \varepsilon_{DD} - \alpha E_y^2\right)E_y = 0, \quad (4)$$

Where, $\varepsilon_{DD} = \varepsilon_1 - \frac{\omega_p^2}{\omega^2}$

The solution of equation (4) is given by

$$E_{y1} = \left(k_1 / k_0\right)\left(2/\alpha\right)^{1/2} \operatorname{sech}\left(k_1\left(z - z_0\right)\right) \quad (5)$$

Where z_0 is the position of the maximum of the field compnent in the nonlinear cover, and $k_1 = k_0\sqrt{\beta^2 - \varepsilon_{DD}}$. The magnetic field components in the nonlinear cover are

$$H_{x1} = i\left(k_1^2 / \omega\mu_0 k_0\right)\left(2/\alpha\right)^{1/2} \operatorname{sech}\left(k_1\left(z - z_0\right)\right)\tanh\left(k_1\left(z - z_0\right)\right), \quad (6)$$

$$H_{z1} = -\left(k / \omega\mu_0\right)E_{y1} \quad (7)$$

In linear dielectric region ($0 < z < t$):

Also, substitution of equations (2) and (3) into Maxwell's equation yields the following linear differential equation to satisfy in the linear dielectric slab

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left(\beta^2 - \varepsilon_2\right)E_y = 0, \quad (8)$$

The solution of equation (8) is given by

$$E_{y2} = A\sinh\left(k_2 z\right) + B\cosh\left(k_2 z\right), \quad (9)$$

where $k_2 = k_0\sqrt{\beta^2 - \varepsilon_2}$. The magnetic field components in the linear dielectric are

$$H_{x2} = -(i/\omega\mu_0)k_2[A\cosh(k_2 z) + B\sinh(k_2 z)], \qquad (10)$$

$$H_{z2} = -(k/\omega\mu_0)[A\sinh(k_2 z) + B\cosh(k_2 z)] \qquad (11)$$

In linear LHM ($z < 0$):

Substituting equations (2) and (3) into Maxwell's equation yields the following linear differential equation to satisfy E_{y3} in the linear LHM slab

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left(\beta^2 - \varepsilon_{eff}^L \mu_{eff}^L\right)E_y = 0, \qquad (12)$$

Thus ε_{eff}^L and μ_{eff}^L are the effective permittivity and permeability of the linear left handed material respectively, and they are given by

$$\varepsilon_{eff}^L = 1 - \frac{\omega_p^2}{\omega^2}, \qquad (13)$$

$$\mu_{eff}^L = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}. \qquad (14)$$

Where ω_p is the plasma frequency, ω_0 is the resonance frequency of the wire, and F is the structure factor. The solution of equation (12) is

$$E_{y3} = Ce^{k_3 z} \qquad (15)$$

where $k_3 = k_0\sqrt{\beta^2 - \mu_{eff}\varepsilon_{eff}}$, A, B, and C are amplitude coefficient determined from the boundary condition, and we can get the magnetic components in the linear LHM as

$$H_{x3} = -(i/\omega\mu_{eff}\mu_0)Ck_3 e^{k_3 z} \qquad (16)$$

$$H_{x3} = -(k/\omega\mu_{eff}\mu_0)Ck_3 e^{k_3 z} \qquad (17)$$

Matching the field component H_x and E_y at the boundaries $z = 0$ and $z = t$, the dispersion relation is

$$\tanh(k_2 t) = \frac{k_1 v - Rk_2}{k_2 - k_1 vR}, \qquad (18)$$

where $R = \frac{k_3}{k_2 \mu_{eff}}$ and $v = \tanh(k_1(z_0 - t))$

Power Flow:

The power flux of the waves propagating in the x-direction is given by

$$P = \frac{1}{2}\int \left(E \times H^*\right) dz = \frac{1}{2}\int E_y H_z dz = P_{NLHM} + P_D + P_{LHM} \qquad (19)$$

Thus P_{NLHM}, P_D and P_{LHM} are stand for the power flux in the nonlinear nonmagnetic LHM, linear dielectric and linear LHM respectively.

$$P_{NLHM} = \frac{1}{2}\left(kk_1 / k_0^2 \omega\mu_0\right)\left(2/\alpha\right)\left[1+v\right] \qquad (20)$$

$$P_D = \frac{1}{2}\left(k/\omega\mu_0\right)B^2\left[P_{1D} + P_{2D} + P_{3D}\right] \qquad (21)$$

$$P_{LHM} = \frac{1}{4}\left(k/k_2\omega\mu_0\mu_{eff}\right)C^2 \qquad (22)$$

where $B = C = k_1/k_0\left(2/\alpha\right)^{1/2}\left(1-v^2\right)^{1/2}\left[R\sinh\left(k_2 t\right) + \cosh\left(k_2 t\right)\right]^{-1}$,

$P_{1D} = R^2\left(\cosh\left(k_2 t\right)\sinh\left(k_2 t\right) - k_2 t\right)/2k_2$,

$P_{2D} = \left(\cosh\left(k_2 t\right)\sinh\left(k_2 t\right) + k_2 t\right)/2k_2$,

and

$P_{3D} = \left(R\sinh\left(k_2 t\right)\right)/k_2$.

Numerical Results and Discussion:

The dispersion relation, equation (18), has been solved numerically to compute the complex effective wave index β as a function of the angular frequency ω, and the power flux for the structure under investigation. The parameters of the nonlinear nonmagnetic LHM, linear dielectric and linear LHM are adjusted that the parameters $\varepsilon_{eff}^{NL}, \varepsilon_{eff}^{L}$, μ_{eff}^{L} are negative in the same frequency range, which lies between $10.2 \sim 10.6\,\mathrm{GHz}$. The parameters were used in carrying out the numerical calculations are: $\omega_p = 2\pi\,\mathrm{GHz}$, $\alpha = 1.55\times10^{-10}\,\mathrm{m^2V^{-2}}$, $\varepsilon_1 = 2.5$, $\varepsilon_2 = 3$, $F = 0.56$, and $\omega_0 = 10.21\,\mathrm{GHz}$.

Figure (3-a) shows the effective refractive index β of the structure versus the angular frequency. It is noticed that the effective refractive index is sensitive to the thickness (t) and increase with the frequency increase. Beside that, when the dielectric medium thickness is increase the effective refractive index decrease. Also, the effective refractive index increase with nonlinearity increase, but when the thickness of the dielectric medium is increased the level of the effective refractive index is decreased as shown in figure (3-b). The slope of the dispersion relation represents the group velocity and has a positive slope which means the power flow in the positive z-direction [12]. The structure behaves like a RHM structure, as if the two

LHM has canceled the effect of each other. The importance of this structure is that controllability on the operating frequency range, and the effective refractive index based on the chosen characteristics of the LHMs.

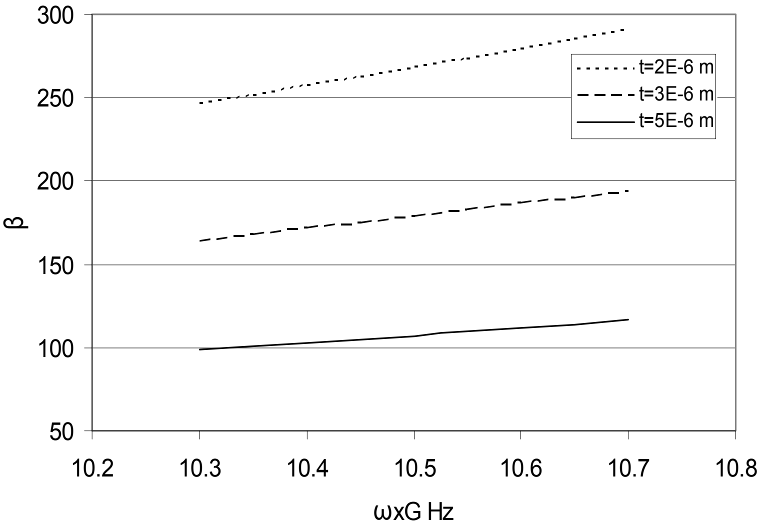

Figure 3: a. the effective nonlinear refractive index versus the angular frequency at nonlinearity v= 0.5, for different dielectric medium thickness.

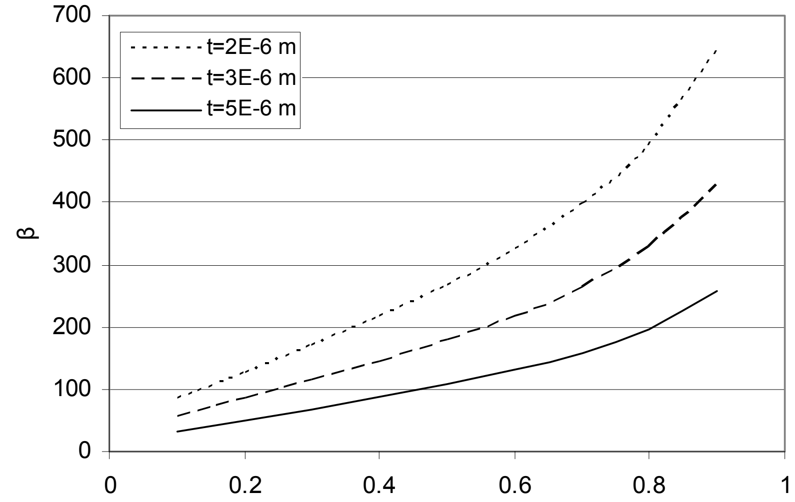

Figure 3-b the effective nonlinear refractive index versus the nonlinearity, at ω = 10.5 G Hz and different thickness.

Figure (4-a), shows the normalized power flow (P/P_0), where $P_0 = \frac{1}{2\alpha\varepsilon_0\omega}$, versus the effective nonlinear refractive index at a constant angular frequency $\omega = 10.5\text{GHz}$. It is noticed that the power flow increases by effective refractive index increase, but the power flow decrease by thickness increase at fixed nonlinearity value of 0.5. In figure 4-b, the power flow increases slightly as the angular frequency increases at fixed nonlinearity of 0.5. As we can see from both figures, figure 4-a, and 4-b the power flow is sensitive to the dielectric medium thickness t.

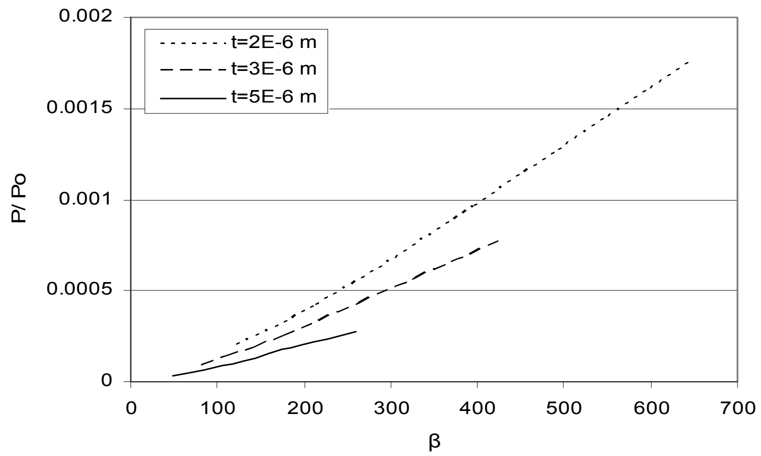

Figure 4: a. the normalized power flux versus the effective nonlinear wave index at ω = 10.5 G Hz, and different thickness.

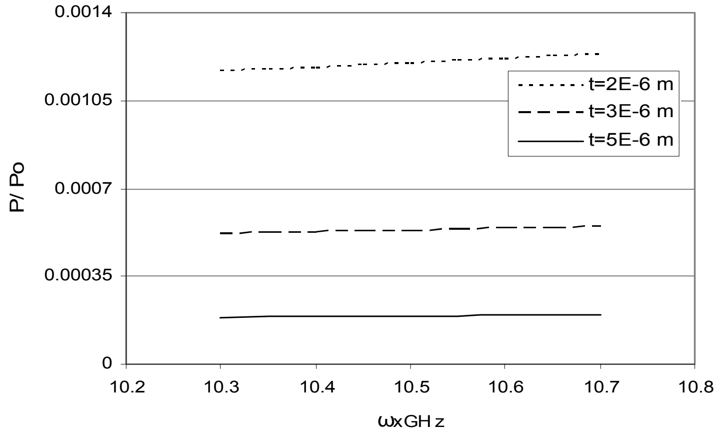

Figure 4-b the normalized power flux versus the angular frequency, at v = 0.5 and different thickness.

Conclusion:

We have studied the nonlinear nonmagnetic LHM of TE surface waves in a three layer waveguide structure; linear dielectric medium sandwiched between LHM cover and nonlinear nonmagnetic LHM as a substrate have been investigated theoretically and numerically. The nonlinear nonmagnetic LHM behaves like a metal with a better advantage, which is one can control its physical characteristics and the operating frequency range. This controllability is not widely available when use certain class of metal compared to LHMs. The dispersion relation was derived to calculate the effective refractive index as a function of the frequency with different thickness of the dielectric medium. We found the effective refractive index is increased with the frequency increase; also the effective refractive index is changing by nonlinearity change, and is sensitive to the dielectric medium thickness. The power flow increases as the effective refractive index increases, also it increases by increasing thickness; and increases slightly with frequency increase in the range, which is $10.2\text{GHz} \sim 10.8\text{GHz}$.

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