Reexamination of Scaling in the Five-dimensional Ising model

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الملخص

إعادة فحص تدريج نموذج ايسنق خماسي الأبعاد

يعتبر نموذج آيسنق الإحصائي ذا أهمية كبيرة، و لذا تم القيام بالعديد من الدراسات التـي تختبر هذا النموذج في أبعاد مختلفة. ولقد وجد أن دراسة هذا النموذج في ثلاثـة أبعـاد أو اقـل توضح تماماً قوانين التدريج للظاهرة الحرجة، وذلك في الأنظمة المحددة والغير محددة.

أما عند دراسة نموذج ايسنق في أربعة أبعاد أو أكثر ، فإن الأمر يصبح أكثر تعقيداً.

تنبأ الباحثون حدوث انحرافاً في شمولية تدريج ايسنق ثلاثـي الأبعـاد، و يحـدث هـذا الانحراف عندما تكون حرارة النظام مساوية لدرجة الحرارة الحرجة التي يتحول عندها النظـام من حالة المغنطة إلي حالة اللا مغنطة، و ذلك على طول خطوط ذات قـيم ثابتـة (A) تخـضع للعلاقة: A=L*L*(T-Tc)/Tc، حيث أن L هو حجم الشبكة البلوريـة، T درجـة الحـرارة المستخدمة، Tc درجة الحرارة الحرجة.

وهناك أبحاث أخرى لم تثبت حدوث هذا الانحراف في شمولية التدريج.

هذه الظاهرة تم التأكد منها باستخدام المحاكاة الحاسوبية بطريقة مونت كارلو.

وبسبب تناقض نتائج الأبحاث المختلفة، لذا فإنه يتم في هذا البحث دراسة إعـادة فحـص نموذج ايسنق خماسي الأبعاد باستخدام المحاكاة الحاسوبية بطريقة مونت كارلو، لذا تـم تطـوير برنامج فورتران لهذا الغرض حول درجة حرارة حرجـة K_c=0.1139150، حيـث وجـد أن

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العلاقات الأسية المختلفة الدالة في حجم النظام L ، تتغير وفقاً لتغيير قيمة الثابت A (موجبـة-صفر–سالبة).

لقد أكدت نتائج المحاكاة الحاسوبية في هذا البحث تنبؤ العالمان (تشن ودوم) فـي حـين اختلفت جزئياً عن نتائج بعض الأبحاث الأخرى.

ABSTRACT

In three dimensions, or more generally, below the upper critical dimension, scaling laws for critical phenomena seem well understood, for both infinite and for finite systems. Above the upper critical dimension (four dimensions and more), finite-size scaling is more difficult.

Deviation was predicted in the universality of the Binder cumulants for three dimensions and more for the Ising model. This deviation occurs if the critical point T = Tc is approached along lines of constant A = L*L*(T-Tc)/Tc, then different exponents which are function of system size L are found depending on whether this constant A is taken as positive, zero, or negative. This effect was confirmed by Monte Carlo simulations. Because of the importance of this effect and the unclear situation in the analogous percolation problem, we reexamine in this article the five-dimensional Glauber kinetics. For this purpose, Monte Carlo simulations of five dimensions Ising models have been investigated by developing a FORTRAN program around a critical point K_c = 0.1139150. Our Simulations confirm the prediction of Chen and Dohm of three different finite-size exponents for the spontaneous magnetization near the critical point which partially contradicts Schulte and Drope findings.

INTRODUCTION:

In recent years, the question of universality of the five–dimensional Ising model has been arisen. This question focuses on the value of susceptibility varying with temperature near the critical temperature for different size of lattices; we will investigate the susceptibility of the five-dimensional Ising model. In 2004 Chen and Dohm predicted theoretically [1], that the widely believed universality principle is violated in the Ising model on the simple cubic lattice with more than only six nearest neighbors. They also found deviations between the theories of dimension of four and more on a Lattice and in the continuum. In 2005 this prediction was partially confirmed [2-3]. Other research groups [4-7] studied the 2D and 3D Ising model for different parameters and also for directed interactions problems occur in the Ising model. Schulte and Drope [3] by Monte Carlo simulations with Glauber [8] and Creutz [9] kinetics, found such violation, but not in the predicted direction. Selke and Shchur [2] tested the square lattice. For this importance effect and the unclear situation in the analogous percolation [10], here we reexamine this universality for the susceptibility ratio and magnetization near the critical point. For this purpose we study first the standard 5D Ising model with ten nearest neighbors.

Our study is based on Monte Carlo simulations for systems with linear different sizes (10, 13, 17, 31, 37, and 71).

We used within this work the critical point: J/kTc = Kc = 0.1139150 as in ref. [11].

A FORTRAN program was developed and used for the above simulation. This program is stated at the appendix (1).

Simulations and Results:

In this article we present new results using Monte Carlo simulation for the universality scaling of five-dimension Ising model with the Susceptibility and the Magnetization along lines of constant near the critical point.

To study the critical behavior of the five dimension Ising Model we define the variable m such as:

$$m = \sum_{i=1}^{N} \frac{\sigma_i}{N} \quad ,$$

where σ_i is the ith spin value of the ith site in the lattice, and N is the total site number of the lattice.

We are interested in the Magnetization and Susceptibility as the following:

$$M = [< |m| >]_{ave}$$

$$\chi = [< m^2 > - < |m| >^2]_{ave}$$

where < --- > stands for a thermodynamic average and [---] $_{ave}$ square brackets for an average over all time.

χ is the Susceptibility, *m* is the magnetization for any iteration and M is the Magnetization of the Ising model over all time.

From our simulation study for different sizes of lattice, by varying the temperature near the critical temperature, the data are shown in table (1).

Table (1) Susceptibility versus temperature with different size size lattices for 10 nearest neighbors

(Tc-T)/Tc	Susceptibility				
	L=10	L=13	L=17	L=23	L=31
-0.05	8.6	8.8	9.2	9.5	9.4
-0.04	11.4	11.1	12.3	12.8	12.2
-0.03	17.4	16.7	16.4	17.6	17.0
-0.02	30.4	24.6	27.6	28.3	27.3
-0.01	64.2	59.6	57.5	56.7	63.5
0	623.0	1129.1	2174.2	3166.4	
0.01	129.0	130.4	122.2	114.0	134.0
0.02	67.9	67.2	64.0	61.8	68.2
0.03	41.7	40.6	43.8	40.0	42.0
0.04	31.3	32.6	31.4	31.8	32.5
0.05	26.9	26.4	25.8	24.4	26.9

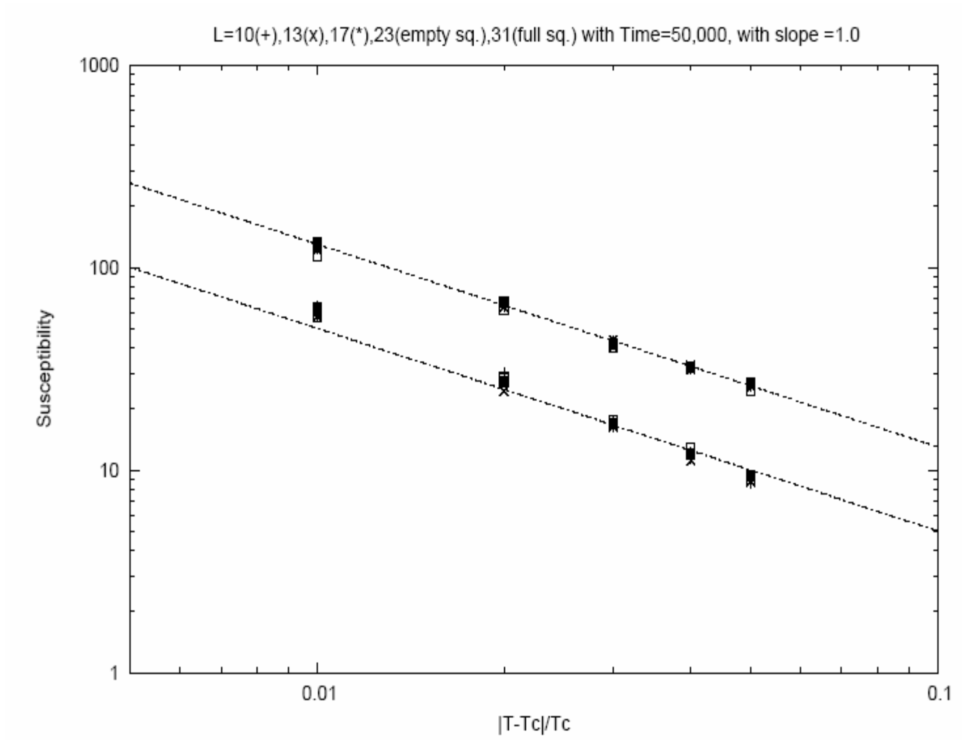


Figure (1): Susceptibility versus temperature with different L's (10, 13, 17, 23, 31), for 10 nearest neighbors as log-log plot, the upper data correspond to T > Tc with amplitude 1.3 , and the lower to T < Tc with amplitude 0.5 , and straight lines had the theoretical slope (-1).

The ratio of susceptibility is calculated by dividing the susceptibility of temperature above Tc to the susceptibility below Tc, then the ratio of susceptibility for |Tc-T|/Tc is obtained as presented in table (2).

Table (2) Ratio of susceptibility versus temperature for different size lattices for 10 neighbors

|Tc-T|/Tc	Ratio of Susceptibility				
	L=10	L=13	L=17	L=23	L=31
0.01	2.0	2.2	2.1	2.0	2.1
0.02	2.2	2.7	2.3	2.2	2.5
0.03	2.4	2.4	2.7	2.3	2.5
0.04	2.8	2.9	2. 6	2.5	2.7
0.05	3.1	3.0	2.8	2.6	2.9

The ratio of susceptibility was drawn versus |Tc-T|/Tc as shown in figure (2).

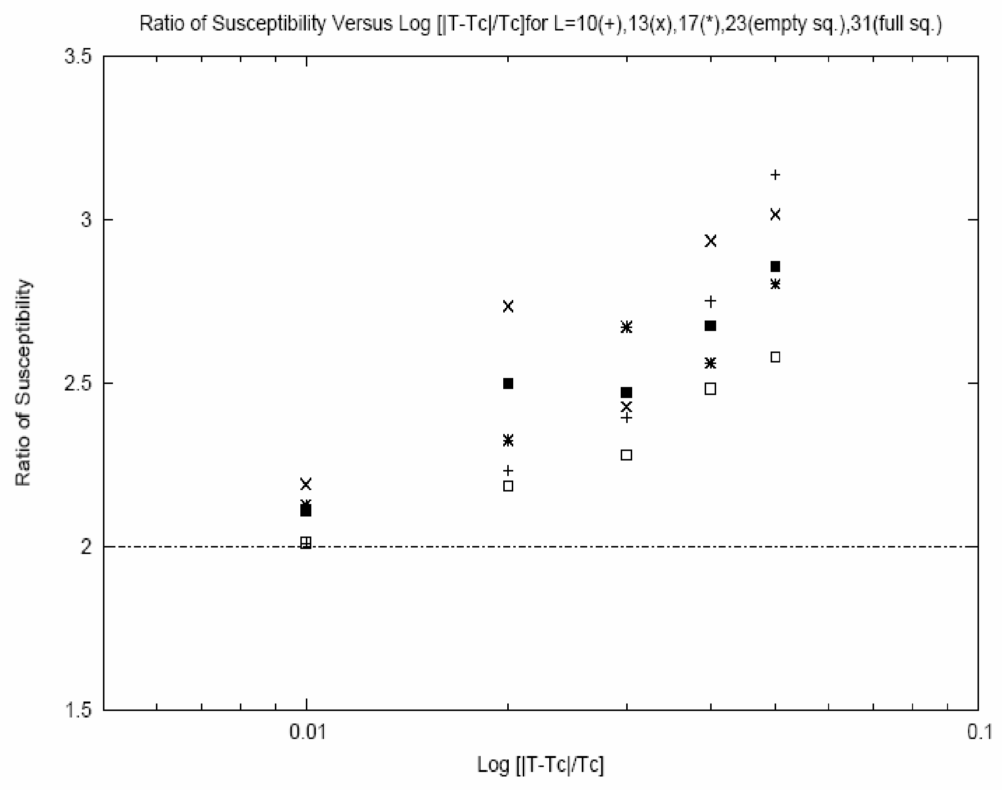


Figure (2): Ratio of susceptibility above to below Tc, plotted semi-logarithmically versus [|Tc-T|/Tc], for the size lattices (10, 13, 17, 23, 31) for 10 neighbors up to time = 50000.

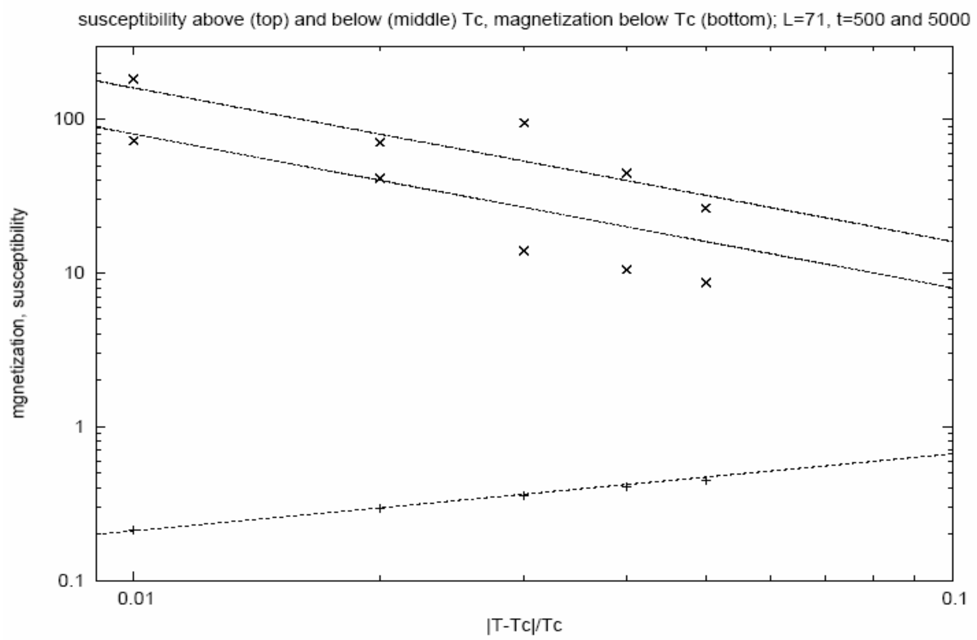
It can be seen that the ratio of susceptibility is roughly constant for varying size of lattice but increases away from the critical temperature.

When large lattice as L=71 is tested for different times (500, 5000), our simulation gives the data as presented in table (3).

Table (3): Magnetization and susceptibility versus temperature with fixed size L = 71 of lattice

L=71 for T=500-5000.

(T-Tc)/Tc	<M>	<M*M>-<M><M> (Susceptibility)	|T-Tc|/Tc
T=500 iterations			
-0.05	0.44731	8.6	0.05
-0.04	0.40531	10.5	0.04
-0.03	0.35609	13.9	0.03
-0.02	0.29553	41.3	0.02
-0.01	0.21340	28.5	0.01
0	0.06993	78126	0
0.01	0.00202	5493	0.01
0.02	0.00017	44.9	0.02
0.03	0.00002	94.7	0.03
0.04	-0.00008	44.7	0.04
0.05	-0.00002	26.4	0.05
T=5000 iterations			
-0.01	0.21337	72.4	0.01
0.01	0.00015	182.3	0.01
0.02	0.00001	70.9	0.02



Figure(3): |M| and susceptibility versus [|Tc-T|/Tc] with fixed size L = 71 of lattice in log-log plot with lines indicating the theoretical slopes -1 and + ½.

It can be seen from figure 3 that susceptibilities scatter much more than the magnetizations.

Now we test the universality of 5D Ising model and vary T along lines of constant A = L*L*(T-Tc)/Tc below, at and above Tc with different L's (10, 13, 17, 23, 31) for many times (500000).

The obtained data are presented in Tables (4 – a,b,c).

Table (4-a): Average magnetization versus different size's L along constant A = L*L*(T-Tc)/Tc =- 1.0 with time = 500,000

L	(T-Tc)/Tc	<|M|>	<M*M>-<M><M> (Susceptibility)	|T-Tc|/Tc	M*M*chi
10	-0.00990	0.20955	64.9	0.00990	2.85
13	-0.00588	0.16424	109.3	0.00588	2.95
17	-0.00345	0.12741	184.1	0.00345	2.99
23	-0.00189	0.09505	358.8	0.00189	3.24
31	-0.00104	0.07121	630.8	0.00104	3.20
31	-0.00104	0.07106	567.2	0.00104	2.86

Table (4-b) : Average magnetization versus different size's L along constant A = L*L*(T-Tc)/Tc =+ 1.0 with time = 500,000

L	(T-Tc)/Tc	<|M|>	<M*M>-<M><M> (Susceptibility)	|T-Tc|/Tc	M*M*chi
10	0.01010	0.02828	124.4	0.01010	0.10
13	0.00595	0.01896	210.0	0.00595	0.08
17	0.00347	0.01262	352.7	0.00347	0.06
23	0.00189	0.00785	622.1	0.00189	0.04
31	0.00104	0.00574	1500.6	0.00104	0.05
31	0.00104	0.00538	1324.3	0.00104	0.04
37	0.00073	0.00426	1789.0	0.00073	0.03

Table (4-c) : Average magnetization versus different size's L along constant A = L*L*(T-Tc)/Tc = 0 with time = 500,000

L	(T-Tc)/Tc	<|M|>	<M*M>-<M><M> (Susceptibility)	|T-Tc|/Tc	M*M*chi
10	0.00000	0.06309	548.8	0.00000	2.18
13	0.00000	0.04565	1077.0	0.00000	2.25
17	0.00000	0.03269	2124.6	0.00000	2.271
23	0.00000	0.02257	4591.8	0.00000	2.34
31	0.00000	0.01458	7299.8	0.00000	1.56

If the average of the absolute value of magnetization is taken, and plotted against the size of lattices for all constants (A = +1, 0, -1) with log-log scale, the slopes are obtained as in figure (4), in agreement with previous theories and simulations [1,8,9].

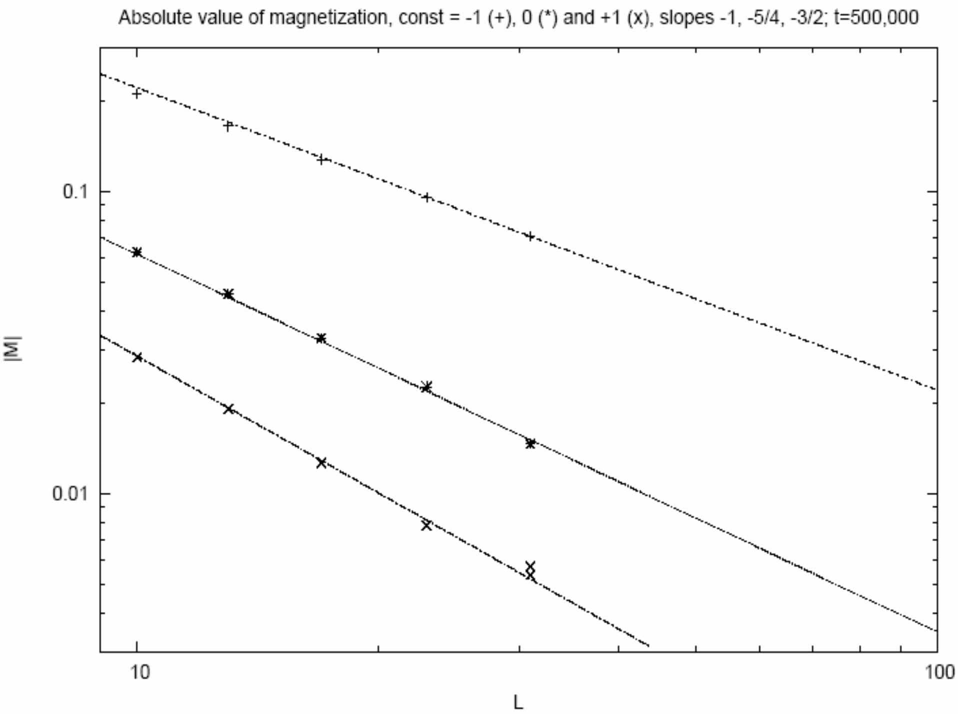


Figure (4) : <|M|> versus size L of lattice (10, 13, 17, 23, 31), in log-log plot along constant A = L*L*(T-Tc)/Tc . The upper data correspond to T<Tc (A=-1) with slope -1, the middle data correspond to T =Tc (A = 0) with slope -5/4, and the lower correspond to T >Tc (A=+1) with slope -3/2.

By drawing the susceptibility versus the size L of lattices for the constants (A = +1, 0, -1) with log-log scale , we get different slopes, twice as large as that for the magnetization in the previous figure, as shown by figure (5).

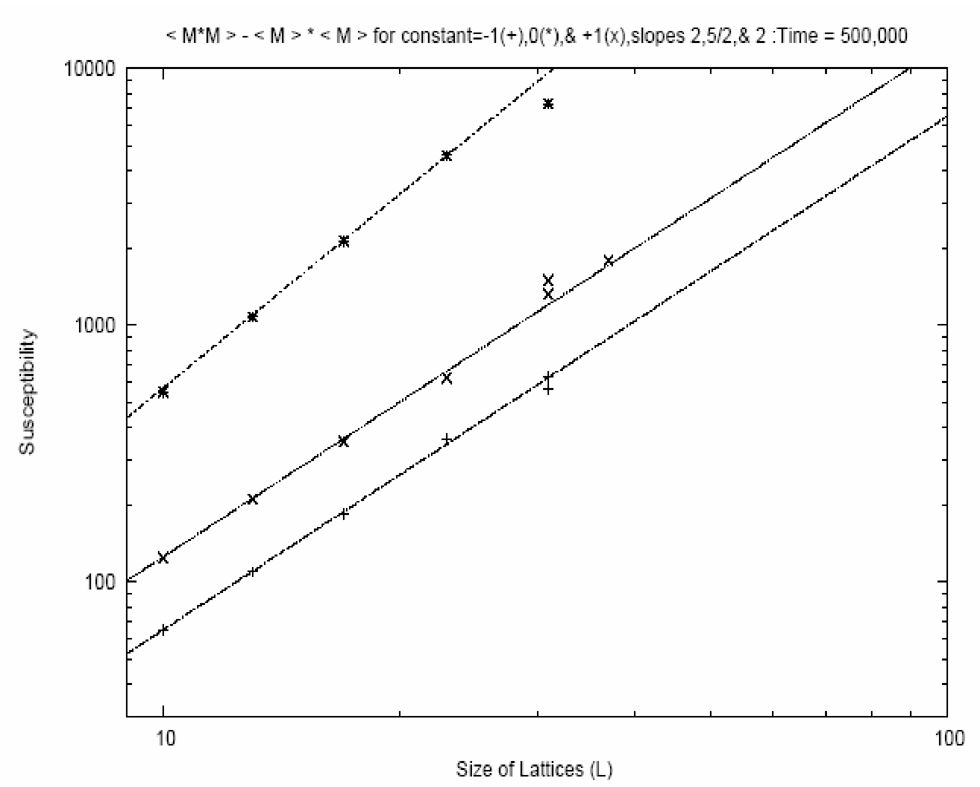


Figure (5): Susceptibility(<M*M> -<M>*<M>) versus size L of lattice (10, 13, 17, 23, 31), as log-log plot along constant A = L*L *(T-Tc)/Tc, the upper data correspond to T=Tc (A=0), the middle data to T > Tc (A=1.0), and the lower data to T < Tc (A=-1). The middle data fit better the indicated slope 2 than the expected slope 3.

CONCLUSION:

This study confirmes [1,8,9] that finite size scaling in high dimensions is described by different exponents if we approach the critical point along different lines in the plane of T-Tc versus 1/(L*L), above, at, and below Tc. This result holds not only for the magnetization [8] but also for the susceptibility. Though the susceptibilities above Tc are problematic.

Appendix
Programming used in Simulations

A: Main program:

```
      PARAMETER(L=17,L2=L*L,L3=L2*L,L4=L2*L2,L5=L3*L2,
     1 LMAX=L5+2*L4)
      INTEGER *8 IBM,IEX
      DIMENSION IEX(-10:10)
      BYTE IS(LMAX)
      DATA TC,MAX,IBM,ISEED/0.113915,500000,1,1/
      IBM=2*ISEED -1
C     TR=1.01
      CONST=0.0
      T=-(TC*CONST/L2)+TC
      T1=TC/T-CONST
C     T= TI*(1.0-0.1/(L*L))
      PRINT *,L,T,T1,MAX,ISEED
C    T = T/TR
C     T=1.01*0.1139150=0.11505415
      LP1=L4+1
      L2PL=L5+L4
      DO 1 I=1,LMAX
1     IS(I)=1
      DO 2  IE=-10,10
      IBM=IBM*16807
      EX=EXP(-2.0*IE*T)
2      IEX(IE)=2147483648.0D0*(4.*EX/(1.0+EX)-2.0)*2147483648.0D0
      DO 3 MC=1,MAX
      DO 4 I=LP1,L2PL
      IE=IS(I)*(IS(I-1)+IS(I+1)+IS(I-L)+IS(I+L)+IS(I-L2)+IS(I+L2)
     1          +IS(I-L3)+IS(I+L3)+IS(I-L4)+IS(I+L4))
      IIBM=IBM*16807
      IF (IBM.LT.IEX(IE)) IS(I)= -IS(I)
      IF(I.NE.2*(L4)+1) GOTO 4
      DO 7 J=1,L4
7      IS(J+L5+L4)=IS(J+L4)
4      CONTINUE
      FACTOR=1.0/(L*L*L*L)
      DO 5 I=1,L4
5      IS(I)=IS(I+L5)
```

```
      MAGN=0
      DO 6 I=LP1,L2PL
6      MAGN=MAGN+IS(I)
       X=MAGN*FACTOR
3      PRINT *,MC,MAGN,X
      STOP
      END
```

B: Analysis program:

```
       INTEGER*8 MAGN,SUMMAG,SUMSQU
       REAL*8 X, AVERGESUMMAG,AVERGESUMSQU
       READ *,L,T,T1,MAX,ISEED
       L5=L*L*L*L*L
       SUMMAG=0
       SUMSQU=0
       COUNT=0
       DO 100 I=1,MAX
       READ *,MC,MAGN
       X=MAGN
       IF(MC.LE.(MAX/2)) GO TO 100
       SUMMAG=SUMMAG+X
C      M = ISUMMAG=ISUMMAG+MAG
        SUMSQU=SUMSQU+X*X
C       M**2= ISUMSQU=ISUMSQU+MAG*MAG
C       PRINT *, MC, ISUMMAG,ISUMSQU
100    CONTINUE
       AVERGESUMMAG=SUMMAG/(MAX*0.5D0)
       AVERGESUMSQU=SUMSQU/(MAX*0.5D0)
       X=AVERGESUMMAG/L5
       CHI=(AVERGESUMSQU-AVERGESUMMAG**2)/L5
        PRINT 1,L,T,X,CHI,ABS(T),X*X*CHI
C      PRINT 1,AVERGESUMMAG,AVERGESUMSQU,
C    1  (AVERGESUMSQU-AVERGESUMMAG**2)/(L*L*L*L*L)
C      SUSCEPITIBILITY=(AVERGESUMSQU-
C      AVERGESUMMAG**2)/(L*L*L*L*L)=<M**2>-
<M>**2/(L*L*L*L*L)
C    1    ISUMSQU*(MAX*0.5D0)-ISUMMAG*ISUMMAG
C    1      FORMAT(1X,3F19.5,I19)
1       FORMAT (1X,I2,5F15.5)
       STOP
       END
```

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