Calculations of the neutrino nucleon cross sections in nuclear matter

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ABSTRACT

In this study, calculations of the total and differential quasi-elastic cross sections for neutrino and antineutrino scattering on nucleons in nuclear matter are presented within the frame of Pauli suppression model by using up to date fits to the nucleon elastic electromagnetic form factors G_E^p, G_E^n, G_M^p, G_M^n, and weak and pseudoscalar form factors.

It was found that the non-zero value of G_E^n has a significant effect on both the total and differential neutrino and antineutrino quasielastic cross sections. We perform a re-analysis of previous neutrino data using updated form factors.

Key words: Scattering; nucleon; neutrino; form factors; quasi-scattering; cross sections

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INTRODUCTION:

Relatively speaking, neutrinos remain elusive particles, only weakly interacting and hard to detect. The interest in neutrinos goes beyond the study of their intrinsic properties and extends to a variety of topics in astro-, nuclear and hadronic physics (Diemoz, *et al*., 2000). Thus, neutrinos serve as a valuable tool in exploring nuclear and hadronic physics issues like, for example, the understanding of the energy production in the sun or of supernova explosions. Neutrinos can probe the interior of objects that otherwise remain inaccessible, in general.

The only way to observe neutrinos is by detecting the secondary particles they create when interacting with matter. Often used targets in neutrino experiments are heavy nuclei, which provide relatively large cross sections (Cristina, 2005). For neutrinos, as well as for antineutrinos, we can distinguish different types of interactions based on particle energy. In the intermediate energy region, the neutrino can scatter from the nucleon as a whole through a charged current (CC) or "quasi-elastic" (QE) ($\nu_{\mu} + N \rightarrow \mu^{-} + N$) or through the neutral current (NC) or "elastic" ($\nu_{\mu} + N \rightarrow \nu + N$) process (Alberico, *et al*, 2000; Paschos, 2002). Charged current scattering is the only practical way towards an understanding of the axial form factors of the nucleon; neutral current scattering, on the other hand, can probe the strange sea quark contribution to the nucleon spin. Above 1 GeV, the target nucleon can be excited to delta state or nucleon resonance (N^{*}), which is referred to as "resonance" or single pion production (Cristina, 2005). At energies even higher than 5 GeV, neutrinos can scatter off leptons or quarks in nucleons. In this case, the neutrino has the resolution to "see" the constituent quarks, and interactions are dominated by neutrino quark scattering, commonly called "deep-inelastic" (DIS) scattering (Bass, 2005; Diemoz, *et al,* 2000). Neutrinos interact with quarks and leptons through the CC interaction which is mediated by W exchange, or through the NC interaction which is mediated by Z exchange (Kusenko, and Weiler, 2002). For detailed and complete reviews of neutrino physics and of recent experimental results the reader is referred to Refs. (Lipari, 2003; Langacker, *et al*., (2005).

In turn, the detailed theoretical understanding of the weak nuclear response is a prerequisite for the analysis of current and future neutrino experiments and a precise knowledge of the neutrino nucleus cross section is therefore essential. Many event generators for neutrino interactions exist, among them are NUANCE, NEUGEN, NEUT and NUX-FLUKA (Gallagher, *et* *al.,* 2005); Zeller, 2003). Commonly they apply impulse approximation and use the Llewellyn-Smith formalism for quasi-elastic events and the Rein-Sehgal model for the resonances (Rein and Sehgal, 1981). The generators differ substantially in how they implement in-medium effects and final state interactions (Hayato, 2002).

Thus far, a tremendous number of theoretical investigations have peen performed using form factors that have been derived on the basis of old experimental data somewhere else (Llewellyn-Smith, 1972). This situation motivates us to re-examine the form factors on the basis on the most recent experimental data and study neutrino scattering on nucleons and nuclei at low and intermediate energies up to about 2 GeV. Our treatment will include the implementation of the in-medium widths. This is an important feature of our study.

The Model:

In order to introduce cross sections, we assume the validity of the so-called impulse approximation. Impulse approximation implies that the incoming particle interacts only with a single nucleon of the nucleus, an assumption adopted already for photo- and electroproduction. Due to the even smaller weak coupling constant, impulse approximation is also applicable for neutrino nucleus scattering. Moreover, the model in this study, pictures the nucleus as a two zero temperature Fermi spheres, one for neutrons and another one for protons (see Figure 1) to ensure a uniform density in momentum space. This means that the whole system can be represented by a Fermi sphere of radius k_F. For nuclei with unequal numbers of neutrons and protons, there are separate radii for protons Fermi sphere k_F^p, for neutrons Fermi sphere k_F^n, and for the whole system k_F. The three radii are related to each other according to (Johnson, and Suwonjaydee, 2004):

$$k_F^3 = \frac{\left(k_F^p\right)^3 + \left(k_F^n\right)^3}{2} \qquad (1)$$

For normalized volumes, the neutron number, the proton number, and the mass number are respectively, as follows: $N = 4\pi\frac{\left(k_F^n\right)^3}{3}$, $Z = 4\pi\frac{\left(k_F^p\right)^3}{3}$, and $A = N + Z = 2\times 4\pi\frac{\left(k_F^3\right)}{3}$.

Aiming at a model which incorporates quasi-elastic scattering we need to find a way of implementing this process simultaneously. For that, we first need to discuss the kinematics of a typical reaction

$$\nu + A \rightarrow l + X \tag{2}$$

where ν is the incident neutrino, A target particle (nucleus), l is the outgoing neutrino, and X is the outgoing particle. For allowed interactions, the nucleon involved must receive enough momentum to position itself outside the relevant Fermi sphere. In order to facilitate the interaction processes, the protons and neutrons, initially, are assumed to populate two concentric spheres as illustrated in Figure1.

The four-momenta of the nucleons p are determined from their initialization. Then the hadronic final state is fully defined: Energy and momentum conservation yields:

$p' = p + q$

Inside the nuclear medium, the following condition has to be fulfilled in the nucleus rest frame:

$$s = M'^2_{eff} = M^2_{eff} - Q^2 + 2E_q M - 2\vec{p}.\vec{q} \tag{3}$$

where Q^2 is the negative of the 4-momentum transfer squared, M is the mass of nucleon, $\vec{p}$ is the Fermi momentum of the nucleon and $\vec{q}$ is the momentum of the exchanged vector boson. This equation further takes into account that the nucleon is bound in a mean field potential which is momentum dependent.

In the nucleon rest frame for $M = M'$, and neglecting the hadronic potentials equation (3) is simplified to

$$s = M^2 = M^2 - Q^2 + 2E_q M - 2\vec{p}.\vec{q} \tag{4}$$

Then only two independent quantities are sufficient to determine the full kinematics. We choose E_ν and Q^2. Further we assume $\vec{q} = q\hat{e}_z$. With that, the system is completely determined and the scattering angle between the neutrino and the charged lepton follows from

$$\cos(\theta) = -\frac{Q^2 - 2E_\nu E_l + m_l^2}{2E_\nu \sqrt{E_l^2 - m_l^2}} \tag{4}$$

with $E_l = E_\nu - E_q$, the neutrino four-vector components can be written as (Lehr, 1999).:

$$\left.\begin{aligned} k_3 &= \frac{\vec{k}.\vec{q}}{q} = \frac{E_\nu^2 - E_\nu\sqrt{E_l^2 - m_l^2}\cos\theta}{q} \\ k_2 &= 0 \\ k_1 &= \sqrt{\left|\vec{k}\right|^2 - k_3^2} \end{aligned}\right\} \quad (5)$$

Let us now introduce the scattering cross sections. The general expression of the quasi-elastic neutrino-nucleon differential cross section for the collision of a neutrino and a nucleon in the nucleon rest frame as a function of the squared momentum transfer, $Q^2 = -q^2$, and the neutrino energy E_ν, is given by (Llewellyn-Smith, 1972):

$$\frac{d\sigma^{free}}{dQ^2} = \frac{M^2 G_F^2 \cos^2(\theta_c)}{8\pi E_\nu^2}\left[A(Q^2) \mp \frac{(s-u)B(Q^2)}{M^2} + \frac{C(Q^2)(s-u)^2}{M^4}\right] \quad (6)$$

here $\frac{d\sigma^{free}(E_\nu)}{dQ^2}$ is the neutrino quasi-elastic differential cross section for free neutron stationary in the lab frame, M is the mass of the nucleon mass, $G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2 = 1.1664\times10^{-5}$ GeV^{-2}. is the Fermi constant, and g_W is the weak coupling strength which has a typical value of 0.7. With $s-u = 4ME_\nu - Q^2 - m_l^2$, $\tau = \frac{Q^2}{4M^2}$, the A, B, and C functions can be rewritten in following general form (Llewellyn-Smith, 1972):

$$A(Q^2) = \frac{(m_l^2 + Q^2)}{M^2}\left\{\left[(1+\tau)F_A^2 - (1-\tau)(F_1^V)^2 + \tau(1+\tau)(F_2^V)^2 + 4\tau F_1^V F_2^V\right] - \frac{m_l^2}{4M^2}\left[\left(F_1^V + F_2^V\right)^2 + \left(F_A + 2F_P\right)^2 - \left(\frac{Q^2}{M^2} + 4\right)F_P^2\right]\right\} \quad (7)$$

$$B = \frac{Q^2}{M^2}F_A\left(F_1^V + F_2^V\right), \quad (8)$$

$$C = \frac{1}{4}\left[F_A^2 + \left(F_1^V\right)^2 + \left(+F_2^V\right)^2\right] \quad (9)$$

Neutrino and antineutrino scattering differ by the sign in front of the B term. The form factors for neutrino and antineutrino scattering are the same because of charge symmetry of the matrix element. With the given dependence on the lepton mass m_l, the cross section is valid for all flavors. Note that F_P has to be multiplied by m_l/M^2 so its contribution is negligible for ν_μ and ν_e, but becomes important for ν_τ.

At this point, the cross section is given in terms of four unknown standard form factors F_A, F_B, F_1^V, and F_2^V. Here F_A is the axial form factor and F_P the pseudoscalar form factor. $F_{1,2}^V$ are the vector form factors. The axial form factor F_A can be can be written in its dipole form as(Stoler, 1993):

$$F_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \qquad (10)$$

where we have for the axial vector constant $g_A = -1.267$ and for the axial mass $M_A^2 = 1.053\,\mathrm{GeV}^2$ (Bernard, *et al.*, 2002). Now the pseudoscalar form factor F_P and the axial form factors F_A can be related (Bernard, *et al.*, 2002):

$$F_P(Q^2) = \frac{2M^2}{\left(Q^2 + m_\pi^2\right)} F_A(Q^2) \qquad (11)$$

Before discussing the vector form factors, let us introduce the so called Sachs form factors $G_m^\nu(Q^2)$ and $G_E^\nu(Q^2)$ Which are determined from the electron scattering process as follows (Stoler, 1993):

$$G_E^\nu(Q^2) = G_E^p(Q^2) - G_E^n(Q^2) \qquad (12)$$

$$G_M^\nu(Q^2) = G_M^p(Q^2) - G_M^n(Q^2) \qquad (13)$$

where G_M and G_E are the magnetic and the electric form factors of the nucleon, respectively.

However, the best analysis as of today is from factors obtained by Bodek (Budd, Bodek, and Arrington, 2003), which takes into account recent electron scattering data from JLAB. (Gayou, O., *et al*) to obtain updated values for the Sachs form factors. With those new vector form factors, they fitted again the old neutrino data and updated also the axial mass which is the largest uncertainty in neutrino nucleon scattering. We will use their set of form factors in this study. The vector form factors can be written in terms of Sachs form factors, the general form can be written as (Budd, Bodek, and Arrington, 2003):

$$F_1^V = \frac{G_E^v(Q^2) + \frac{Q^2}{4M^2} G_M^v(Q^2)}{\left(1 + \frac{Q^2}{4M^2}\right)} \tag{14}$$

and

$$F_2^V = \frac{G_M^v(Q^2) - G_E^v(Q^2)}{\left(1 + \frac{Q^2}{4m^2}\right)} \tag{15}$$

The simplest parameterization of the form factors used so far is the dipole form factor for many of the neutrino experiments. This type of parameterization has assumed the form factors are of dipole type approximation,

$$\mathrm{G_D}(Q^2) = \frac{1}{\left(1 + \frac{\mathrm{Q}^2}{\mathrm{M}_\nu^2}\right)^2} \tag{16}$$

where $\mathrm{M}_\nu^2 = 0.71\,\mathrm{GeV}^2$.

Using the well known magnetic moments of the proton μ_p = 2.793 and of the neutron μ_n = .1.913, Sachs form factors can be expressed in terms of the dipole form factor as (Stoler, 1993):

$$\mathrm{G_E^p}(\mathrm{Q}^2) = \mathrm{G_D}(Q^2),\ \mathrm{G_E^n}(Q^2) = 0 \tag{17}$$

$$\mathrm{G_M^p}(\mathrm{Q}^2) = \mu_p \mathrm{G_D}(Q^2),\ \mathrm{G_M^n}(Q^2) = \mu_n G_D(Q^2) \tag{18}$$

For the electric form factor of the neutron we take the following parameterization (Krutov, and Troitsky., (2003):

$$\mathrm{G}_E^n(Q^2) = -\mu_n \frac{\mathrm{a}\tau}{(1+b\tau)} \mathrm{G_D}(Q^2) \tag{19}$$

Having now a complete formalism and a state-of-the-art parametrization of the form factors we can study the charged current cross sections. Now, expressions (10-19) can be substituted into equations (7-9) for completing the definition the functions A, B, and C. Now, initializing neutrino events at every test particle and after checking for Pauli blocking, the cross section is calculated according to According to Randy's Pauli suppression model, the nucleus effective differential cross section per neutron is calculated as the following :

$$\frac{d\sigma_{\nu A\to lX}}{dQ^2} = 4\int_0^{k_F} \frac{d^3k_n}{(2\pi\hbar)^3}\frac{d\sigma^{free}_{\nu A\to lX}}{dQ^2} \tag{20}$$

The factor 4 is introduced to take into account the spin and isospin states of a nucleon. The cross section per neutron, is given by (Johnson, and Suwonjaydee, 2004):

$$\frac{d\sigma(E_\nu)}{dQ^2} = \frac{4}{A-Z}\int_0^{k_F} d^3k_n \frac{f\left(E_\nu, Q^2, \vec{k}_n\right)}{(2\pi\hbar)^3}\frac{d\sigma^{free}}{dQ^2} \tag{21}$$

where k_F is the Fermi momentum of the nucleus, and $f\left(E_\nu, Q^2, \vec{k}_n\right)$ is either 1 or 0. This choice is motivated by the vacuum quasi-elastic scattering which is only possible on neutrons $\nu_\mu n \to \mu^- p$ and therefore allows a direct observation of in-medium effects since this cancels differences due to the proton-neutron number ratio in different nuclei (Effenberger, (1999).

Integration of dσ/dQ^2 over Q^2 yields the total cross section. Thus,

$$\sigma(E_\nu) = \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{d^2\sigma}{dQ^2} \tag{22}$$

where

$$\begin{aligned} Q^2_{\min} &= -m_l^2 + 2E_\nu(E_l - |\vec{k}'| \\ &= \frac{2E_\nu^2 M - Mm_l^2 - E_\nu m_l^2 - E_\nu\sqrt{\left(s-m_l^2\right)^2 - 2\left(s+m_l^2\right)^2 M^2 + M^4}}{2E_\nu + M}, \end{aligned} \tag{23}$$

$$\begin{aligned} Q^2_{\max} &= -m_l^2 + 2E_\nu(E_l + |\vec{k}'| \\ &= \frac{2E_\nu^2 M - Mm_l^2 + E_\nu m_l^2 + E_\nu\sqrt{\left(s-m_l^2\right)^2 - 2\left(s+m_l^2\right)^2 M^2 + M^4}}{2E_\nu + M} \end{aligned} \tag{24}$$

and

$$s = M^2 + 2ME_\nu \tag{25}$$

III. Numerical calculations and results:

In this study we have calculated the differential and total inclusive cross section for CC quasi-elastic scattering. For a neutrino quasi-elastic scattering off event, we can consider the entire neutron sphere being shifted an amount of *q* as shown in Figure 2. For quasi-elastic scattering of neutrinos off shifted by stationary neutrons, the magnitude of the 3-mometum transfer, $|\vec{q}|$, can be written as:

$$|\vec{q}| = \sqrt{Q^2 + \frac{\left(Q^2 + M_p^2 - M_n^2\right)^2}{4M_n^2}} \quad (26)$$

where Q^2 is the negative of the 4-momentum transfer squared, and M_p and M_n are the masses of a proton and a neutron.

A possible solution is to assume three independent kinematical quantities to be known. In addition to E_ν and Q^2 we, require, for instance, E_l to be fixed. It is worth mentioning here that the third independent kinematical quantity was only required to account for the potentials. Then we can calculate specific cross sections fixed by the neutrino energy and the properties of the outgoing lepton. Having one degree of freedom less, the numerical effort is significantly reduced. In this study, we have used $\vec{k}_F$ = 225 MeV and E_B = 25 MeV for nucleus, taken from an electron scattering analysis.

The differential cross section for various values of the neutrino energy for the reaction $\nu_\mu\, n \rightarrow \mu^-\, p$ is calculated using equation (20) and displayed in Figure 3. The in-medium CC quasi-elastic differential cross section per neutron is calculated from equation (21) and is plotted in Figure 4 for a reaction of muon neutrinos on different nuclei for different neutrino energies. ”In-medium” denotes here the calculation including Pauli blocking and the vacuum calculation is shown by the dashed line. For low momentum transfers up to about 0.2 , it was found that the cross section is reduced significantly due to Pauli blocking. The cross section on the nucleus, which follows from equation (22), is integrated over all momenta, which averages those effects. Relatively speaking, the reduction for low Q^2 is a consequence of Pauli blocking.

The equivalent plot for the total cross section is shown in Figure 5. The total reduction of the cross section as an effect of Pauli blocking is of the order of about 10-15 % slightly depending on the nucleus. For comparison, the long-dashed line shows the elementary cross section. In the framework presented here, the obtained results agree nicely with the experimental data as well as with other calculations (Budd, Bodek and Arrington, 2005; Paschos and Yu, 2002).

SUMMARY:

We have presented a simplified model for quasi-elastic neutrino nucleons as well as nuclei scattering, where we neglected the hadronic potentials. We perform a re-analysis of previous neutrino data using updated form factors. The model developed in the framework of this work should be able to describe neutrino reactions on both nucleons and nuclei at intermediate energies of interest for future neutrino experiments. We perform a re-analysis of previous neutrino data using updated form factors.

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Figure Captions:

Figure 1	Concentric Fermi spheres in momentum space for protons (red or inside) and neutrons(blue or outside). Plots from Notes on Llewellyn Smith Model of Fermi Motion Paper.
Figure 2	Neutron sphere (blue or 1) displaced from proton sphere (red or 2) by the magnitude of the 3 vector part of the momentum transfer. The overlap region (hatched, labeled *D*) is the volume of neutrons excluded from interacting by the Pauli principle. Plots from Notes on Llewellyn Smith Model of Fermi Motion Paper.
Figure 3	Differential cross section for $\nu_\mu n \to \mu^- p$.
Figure 4	In-medium effects on the inclusive quasi-elastic differential cross section for $\nu_\mu n \to \mu^- p$ per neutron and for different nuclei, Eν = 1.0 GeV.
Figure 5	In-medium effects on the inclusive quasi-elastic total cross section for different nuclei.

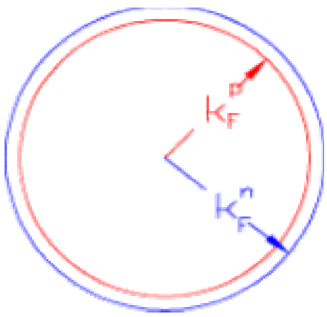

Figure 1

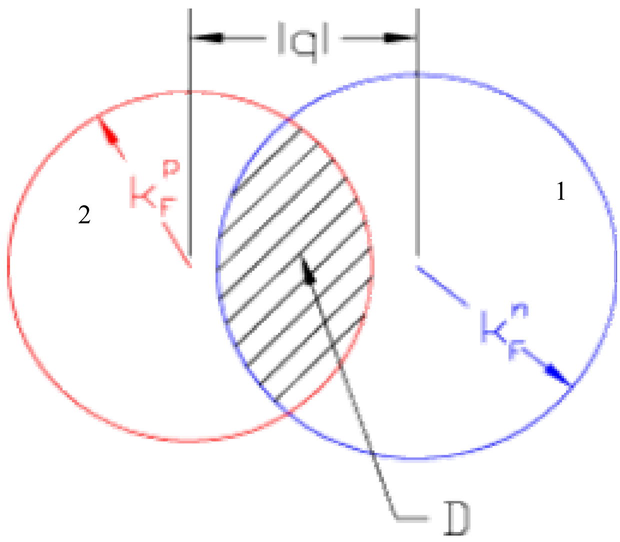

Figure 2

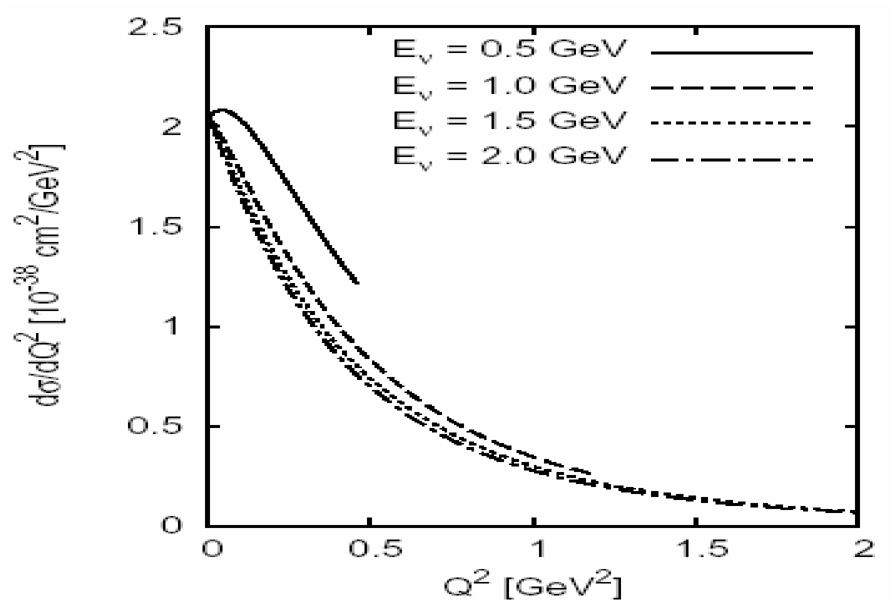

Figure 3

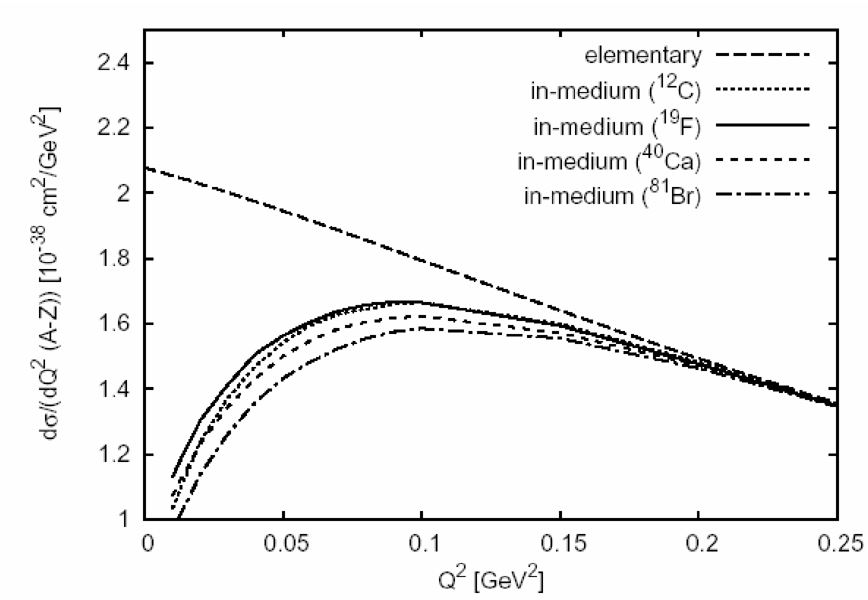

Figure 4

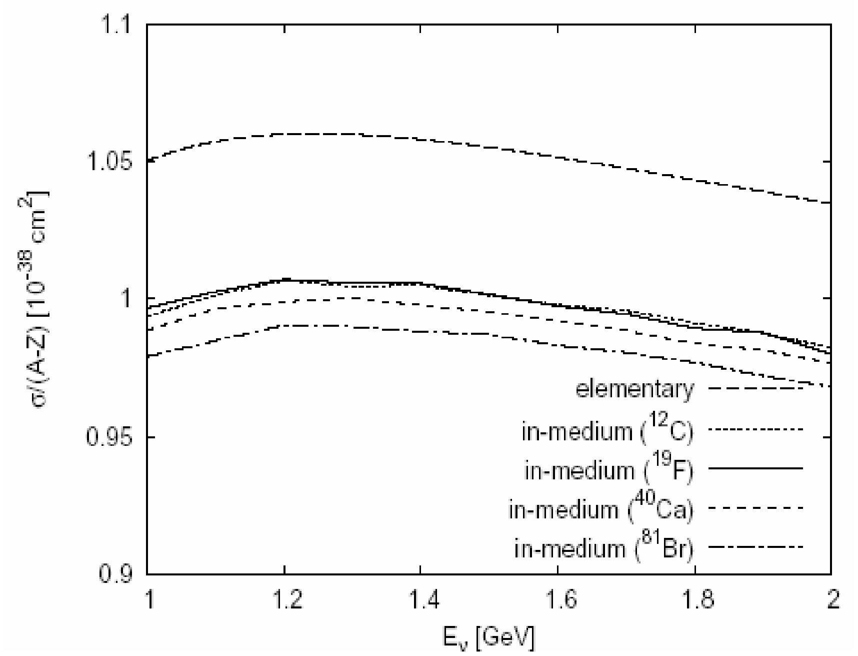

Figure 5